3.5 Hydraulic Gradient and Energy Gradient. The hydraulic grade line, or the hydraulic gradient, in open flow is the water surface, and in pipe flow it connects the elevations to which the water would rise in piezometer tubes along the pipe. The energy gradient is at a distance equal to the velocity head above the hydraulic gradient. In both open and pipe flow the fall of the energy gradient for a given length of channel or pipe represents the loss of energy by friction. When considered together, the hydraulic gradient and the energy gradient reflect not only the loss of energy by friction, but also the conversions between potential and kinetic energy.

In the majority of cases the end objective of hydraulic computations relating to flow in open channels is to determine the curve of the water surface. These problems involve three general relationships between the hydraulic gradient and the energy gradient. For uniform flow the hydraulic gradient and the energy gradient are parallel and the hydraulic gradient becomes an adequate basis for the determination of friction loss, since no conversion between kinetic and potential energy is involved. In accelerated flow the hydraulic gradient is steeper than the energy gradient, and in retarded flow the energy gradient is steeper than the hydraulic gradient. An adequate analysis of flow under these conditions cannot be made without consideration of both the energy gradient and the hydraulic gradient.

4. Open Channel Flow

4.1 Steady, Unsteady, Uniform, and Nonuniform Flow. Steady flow exists when the discharge passing a given cross section is constant with respect to time. The maintenance of steady flow in any reach requires that the rates of inflow and outflow be constant and equal. When the discharge varies with time, the flow is unsteady.

Service work will involve problems in unsteady flow in the analysis of discharge from conduits and spillways and in natural and improved channels where discharge varies during periods of runoff.

Steady flow includes two conditions of flow; uniform and nonuniform. Flow is steady and uniform when the mean velocity and the cross-sectional area are equal at all sections in a reach. Flow is steady and nonuniform when either the mean velocity or the cross-sectional area or both vary from section to section.

4.2 Elements of Cross Sections. The elements of cross sections required for hydraulic computations are:

- a, the cross-sectional area of flow;
- p, the wetted perimeter; that is, the length of the perimeter of the cross section in contact with the stream;
- $r = a/p$, the hydraulic radius, which is the cross-sectional area of the stream divided by the wetted perimeter.

General formulas for determining area, wetted perimeter, hydraulic radius, and top width in trapezoidal, rectangular, triangular, circular, and parabolic sections are given by drawing ES-33.

4.3 Manning's Formula. The most widely used open channel formulas express mean velocity of flow as a function of the roughness of the channel, the hydraulic radius, and the slope of the energy gradient. They are empirical equations in which the values of constants and exponents have been derived from experimental data. Manning's formula is one of the most widely accepted and commonly used of the open channel formulas:

$$v = \frac{1.486}{n} r^{2/3} s^{1/2} \qquad (5.4\text{-}1)$$

v = mean velocity of flow in ft. per sec.
r = hydraulic radius in ft.
s = slope of the energy gradient.
n = coefficient of roughness.

Manning's formula gives values of velocity consistent with experimental data and closely comparable to those computed by the Kutter-Chezy formula. For very flat slopes the Kutter-Chezy formula is considered to be preferable by some authorities. The Manning formula has the advantage of simplicity. The alignment chart, drawing ES-34, may be used to solve for v, r, s, and n when any three are known.

4.4 Coefficient of Roughness, n. The Manning formula is expressed so as to use the same n as is used in the Kutter formula. Table 93, p. 287, "King's Handbook", compares values of n which will make the Kutter-Chezy and Manning formulas equivalent. This table and many other comparisons between the two formulas show that Kutter's n need not be modified for use in Manning's formula when the slope is equal to or greater than 0.0001 and the hydraulic radius is between 1.0 and 20 or 30 feet.

The computed discharge for any given channel or pipe will be no more reliable than the value of n used in making the computation. The engineer, when he is selecting the value of n, is in fact estimating the resistance to flow of a given channel or pipe. This estimate affects the design discharge capacity and the cost and, therefore, requires careful consideration.

In the case of pipes and lined channels this estimate is easier to make, but it should be made with care. A given situation will afford specific information on such factors as size and shape of cross section, alignment of the pipe or channel, type and condition of the material forming the wetted perimeter. Knowledge of these factors, associated with the published results of experimental investigations and experience, make possible selections of n values within reasonably well-defined limits of probable error.

Natural channels and excavated channels, subject to various types and degrees of change, present a more difficult problem. The selection of appropriate values for design of drainage, irrigation, and other excavated channels is covered by handbook data relating to those subjects.

The value of n is influenced by several factors; those exerting the greatest influence are:

(1) The physical roughness of the bottom and sides of the channel. The types of natural material forming the bottom and sides and the degree of surface irregularity are the guides to evaluation. Soils made up of fine particles on smooth, uniform surfaces result in relatively low values on n. Coarse materials such as gravel or boulders and pronounced surface irregularity cause the higher values of n.

(2) Vegetation. The value of n should be an expression of the retardance to flow, as it will be affected by height, density, and type of vegetation. Consideration should be given to density and distribution of the vegetation along the reach and the wetted perimeter; the degree to which the vegetation occupies or blocks the cross-sectional area of flow at different depths; the degree to which the vegetation may be bent or "shingled" by flows of different depths.

(3) Variations in size and shape of cross section. Gradual and uniform increase or decrease in cross section size will not significantly affect n, but abrupt changes in size or the alternating of small and large sections call for the use of a somewhat larger n. Uniformity of cross-sectional shape will cause relatively little resistance to flow; whereas variation, particularly if it causes meandering of the major part of the flow from side to side of the channel, will increase n.

(4) Channel alignment. Curvature on relatively large radii and without frequent changes in direction of curvature will offer comparatively low resistance to flow. Severe meandering with the curves having relatively small radii will significantly increase n.

(5) Silting or scouring. Whether either or both of these processes are active and whether they are likely to continue or develop in the future is important. Active silting or scouring, since they result in channel variation of one form or another, will tend to increase n.

(6) Obstructions. Log jams and deposits of any type of debris will increase the value of n; the degree of effect is dependent on the number, type, and size of obstructions.

The value of n, in a natural or constructed channel in earth, varies with the season and from year to year; it is not a fixed value. Each year n increases in the spring and summer, as vegetation grows and foliage develops, and diminishes in the fall as the dormant season develops. The annual growth of vegetation, uneven accumulation of sediment in the channel, lodgment of debris, erosion and sloughing of banks, and other factors all tend to increase the value of n from year to year until the hydraulic efficiency of the channel is improved by clearing or clean-out.

All of these factors should be studied and evaluated with respect to kind of channel, degree of maintenance, seasonal requirements, and other considerations as a basis for making a determination of n. As a general guide to judgment, it can be accepted that conditions tending to induce turbulence will increase retardance; and those tending to reduce turbulence will reduce retardance. Table 5.4-1 lists values of n taken from various sources which will be useful as a guide to the value to be used in a given case.

TABLE 5.4-1. VALUES OF ROUGHNESS COEFFICIENT, n

Type of Conduit and Description	Values of n			References
	Min.	Design	Max.	
Pipe				
Cast-iron, coated	0.010	0.012 - 0.014	0.014	1
Cast-iron, uncoated	0.011	0.013 - 0.015	0.015	1
Wrought iron, galvanized	0.013	0.015 - 0.017	0.017	1
Wrought iron, black	0.012		0.015	1
Steel, riveted and spiral	0.013	0.015 - 0.017	0.017	1
Corrugated	0.021	0.025	0.0255	2
Wood stave	0.010	0.012 - 0.013	0.014	1
Neat cement surface	0.010		0.013	1
Concrete	0.010	0.012 - 0.017	0.017	1,6
Vitrified sewer pipe	0.010	0.013 - 0.015	0.017	1
Clay, common drainage tile	0.011	0.012 - 0.014	0.017	1
Lined Channels				
Metal, smooth semicircular	0.011		0.015	1,5
Metal, corrugated	0.0228	0.024	0.0244	2
Wood, planed	0.010	0.012	0.015	1,5
Wood, unplaned	0.011	0.013	0.015	1,5
Neat cement-lined	0.010		0.013	1,5
Concrete	0.012	0.014 - 0.016	0.018	1,5
Cement rubble	0.017		0.030	1,5
Vegetated, small channels, shallow depths				
Bermuda grass; long - 13", green	0.042			3
Long - 13", dormant	0.035		0.28	3
Short - 3", green	0.034			3
Short - 3", dormant	0.034			3
Sericea Lespedeza; long -16", green	0.076		0.22	3
Long - 16", dormant	0.050			3
Short - 2", green	0.033			3
Short - 2", dormant	0.034			3
Unlined Channels				
Earth; straight and uniform	0.017	0.0225	0.025	1
Dredged	0.025	0.0275	0.033	1
Winding and sluggish	0.0225	0.025	0.030	1
Stony bed, weeds on bank	0.025	0.035	0.040	1
Earth bottom, rubble sides	0.028	0.030 - 0.033	0.035	1

(Continued on next page)

TABLE 5.4-1 (Continued). VALUES OF ROUGHNESS COEFFICIENT, n

Type of Conduit and Description	Values of n			References
	Min.	Design	Max.	
Unlined Channels—Continued				
Rock cuts; smooth and uniform	0.025	0.033	0.035	1
Jagged and irregular	0.035		0.045	1
Natural Streams				
(1) Clean, straight banks, full stage, no rifts or deep pools	0.025		0.033	1,4
(2) Same as (1) but more weeds and stones	0.030		0.040	1,4
(3) Winding, some pools and shoals, clean	0.033		0.045	1,4
(4) Same as (3), lower stages, more ineffective slopes and sections	0.040		0.055	1,4
(5) Same as (3), some weeds and stones	0.035		0.050	1,4
(6) Same as (4), stony sections	0.045		0.060	1,4
(7) Sluggish reaches, rather weedy, very deep pools	0.050		0.080	1,4
(8) Very weedy reaches	0.075		0.150	1,4

REFERENCES:

1. "King's Handbook", pp. 182 and 268.
2. "Hydraulics of Corrugated Metal Pipes" by H. M. Morris, St. Anthony Falls Hydraulic Laboratory, University of Minnesota.
3. "Flow of Water in Channels Protected by Vegetative Linings" by W. O. Ree and V. J. Palmer; and USDA Technical Bulletin No. 967, February 1949.
4. "Low Dams" by National Resources Committee, U. S. Government Printing Office, Washington, D. C., pp. 227-233.
5. "The Flow of Water in Flumes" by Fred C. Scobey; USDA Technical Bulletin No. 393, Dec. 1933.
6. "Hydraulic Studies of Twenty-four Inch Culverts", studies by St. Anthony Falls Hydraulic Laboratory, University of Minnesota; The American Concrete Pipe Association; and the Portland Cement Association.
7. "The Flow of Water in Irrigation Channels" by Fred C. Scobey, USDA Bulletin 194, 1914.
8. "Flow of Water in Drainage Channels" by C. E. Ramser, USDA Technical Bulletin No. 129, 1929.
9. "Some Better Kutter's Formula Coefficients" by R. E. Horton, Engineering News, February 24, May 4, 1916.

4.5 Critical Flow. Critical flow is the term used to describe open channel flow when certain relationships exist between specific energy and discharge and between specific energy and depth of flow. Specific energy is the total energy head at a cross section measured from the bottom of the channel. The conditions described as critical flow are those which exist when the discharge is maximum for a given specific energy head, or stated conversely, those which exist when the specific energy head is minimum for a given discharge.

Consider the specific energy and discharge at a section in any channel, using the notation

Q = total discharge.
q = Q/T = discharge per unit width of channel.
a = cross-sectional area of flow.
d = depth of flow to the bottom of the section.
d_m = a/T = mean depth of flow.
T = top width of the stream.
v = mean velocity of flow.
g = acceleration of gravity.
H_e = specific energy head, i.e., the energy head referred to the bottom of channel.

The specific energy head (see Fig. 5.3-1) is:

$$H_e = d + \frac{v^2}{2g}$$

From equation (5.3-2) $v = Q/a$; therefore,

$$H_e = d + \frac{Q^2}{2ga^2} \qquad (5.4\text{-}2)$$

By solving this equation for the H_e at which Q is a maximum or the depth at which H_e is a minimum, the following general equation for critical flow in any channel may be obtained. (See "King's Handbook", pp. 372-373):

$$\frac{Q^2}{g} = \frac{a^3}{T} \qquad (5.4\text{-}3)$$

From equation (5.4-3) $Q^2/a^2 = ag/T$; and since $Q^2/a^2 = v^2$ and $a = d_m T$, the specific energy when flow is critical is:

$$H_e = d + \frac{a}{2T} = d + \frac{d_m}{2} \qquad (5.4\text{-}4)$$

Study of the specific energy diagram on drawing ES-35 will give a more thorough understanding of the relationships between discharge, energy, and depth when flow is critical. While studying this diagram, consider the following critical flow terms and their definitions:

Critical discharge - The maximum discharge for a given specific energy, or a discharge which occurs with minimum specific energy.

Critical depth - The depth of flow at which the discharge is maximum for a given specific energy, or the depth at which a given discharge occurs with minimum specific energy.

Critical velocity - The mean velocity when the discharge is critical.

Critical slope - That slope which will sustain a given discharge at uniform, critical depth in a given channel.

Subcritical flow - Those conditions of flow for which the depth is greater than critical and the velocity is less than critical.

Supercritical flow - Those conditions of flow for which the depth is less than critical and the velocity is greater than critical.

The curves show the variation of specific energy with depth of flow for several discharges in a channel of unit width. These curves are plotted from the equation, $H_e = d + (q^2 \div 2gd^2)$, by taking constant values of q, assuming d, and computing H_e. Similar curves for any discharge at a section of any form may be obtained from the general equation (5.4-2). Certain points, as illustrated by these curves, should be noted:

(a) There is a different critical depth for every discharge. In this graph all critical depths fall on the line defined by the equation $H_e = 3d_c/2$; in the general case, critical depths will fall on a curve defined by equation (5.4-4).

(b) In a specific energy diagram the pressure head and velocity head are shown graphically. The pressure head, depth in open channel flow, is represented by the horizontal scale as the distance from the vertical axis to the line along which $H_e = d$. The velocity head at any depth is represented by the horizontal distance from the line along which $H_e = d$, to the curve of constant q.

(c) For any discharge there is a minimum specific energy, and the depth of flow corresponding to this minimum specific energy is the critical depth. For any specific energy greater than this minimum there are two depths, sometimes called alternate stages, of equal energy at which the discharge may occur. One of these depths is in the subcritical range and the other is in the supercritical range.

(d) At depths of flow near the critical for any discharge, a minor change in specific energy will cause a much greater change in depth.

(e) Through the major portion of the subcritical range the velocity head for any discharge is relatively small when compared to specific energy, and changes in depth are approximately equal to changes in specific energy.

(f) Through the supercritical range the velocity head for any discharge increases rapidly as depth decreases; and changes in depth are associated with much greater changes in specific energy.

In addition to its importance in the discharge-energy relationship, critical velocity has significance as the velocity with which gravity waves travel in relatively shallow water. If, in the general equation (5.4-3) Q = av and the appropriate values for a channel of unit width are substituted, the critical velocity is found to be $\sqrt{gd_c}$. The velocity of propagation of gravity waves in shallow water is also $\sqrt{gd}$, d being the depth of water. Therefore, a wave may be propagated upstream in subcritical flow but not in supercritical flow.

4.5.1 General Formulas for Critical Flow. General formulas for critical flow in any section are:

$$\frac{Q^2}{g} = \frac{a^3}{T} \tag{5.4-3}$$

$$H_e = d_c + \frac{d_m}{2} \tag{5.4-4}$$

$$d_m = \frac{v_c^2}{g} \tag{5.4-5}$$

$$d_m = \frac{Q_c^2}{a^2 g} \tag{5.4-6}$$

$$v_c = \sqrt{gd_m} \tag{5.4-7}$$

$$Q_c = a\sqrt{gd_m} \tag{5.4-8}$$

Symbols used in these formulas are:

H_e = specific head.

Q_c = critical discharge.

$q_c = Q_c/T$ = critical discharge per unit width of channel.

a = cross-sectional area.

T = width of water surface.

d_c = critical depth.

$d_m = a/T$ = mean depth of critical flow.

v_c = critical velocity.

g = acceleration of gravity.

See drawing ES-33 for the symbols for channel dimensions.

4.5.2 Critical Flow Formulas for Rectangular Channels. See paragraph 4.5.1 for the symbols used in the following formulas:

$$H_e = 3/2\ d_c \qquad (5.4\text{-}9)$$

$$d_c = 2/3\ H_e \qquad (5.4\text{-}10)$$

$$d_c = \frac{v_c^2}{g} \qquad (5.4\text{-}11)$$

$$d_c = \sqrt[3]{\frac{q_c^2}{g}} \qquad (5.4\text{-}12)$$

$$d_c = \sqrt[3]{\frac{Q_c^2}{b^2 g}} \qquad (5.4\text{-}13)$$

$$v_c = \sqrt{g d_c} \qquad (5.4\text{-}14)$$

$$v_c = \sqrt[3]{g q_c} \qquad (5.4\text{-}15)$$

$$v_c = \sqrt[3]{\frac{g Q_c}{b}} \qquad (5.4\text{-}16)$$

$$q_c = d_c^{3/2} \sqrt{g} \qquad (5.4\text{-}17)$$

$$Q_c = 5.67\ b d_c^{3/2} \qquad (5.4\text{-}18)$$

$$Q_c = 3.087\ b H_e^{3/2} \qquad (5.4\text{-}19)$$

Graphical solutions for Q_c or d_c in equation (5.4-18) may be made by the use of the alignment chart on drawing ES-24.

4.5.3 Critical Flow Formulas for Trapezoidal Channels. See paragraph 4.5.1 for the symbols used in the following formulas:

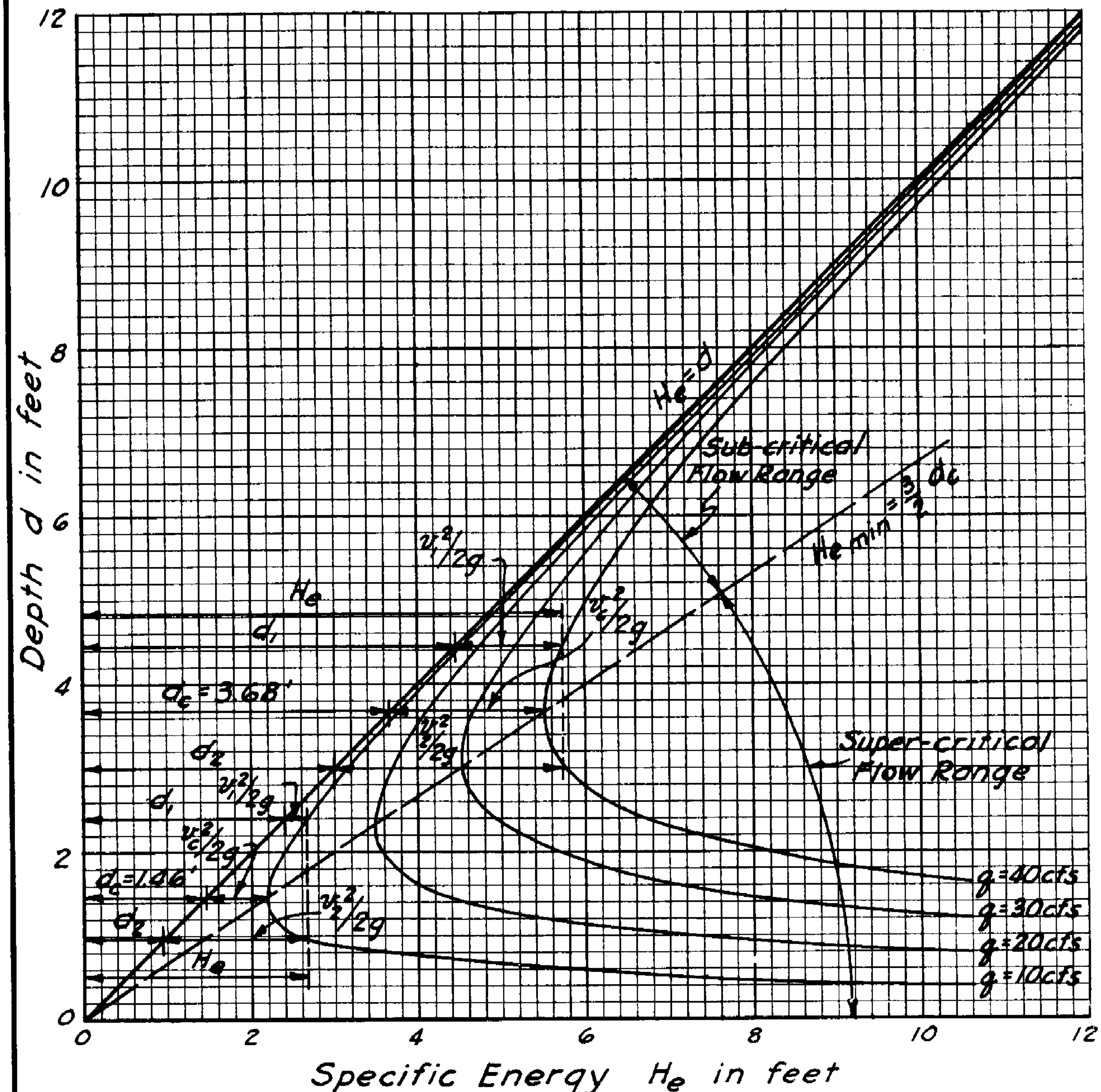
$$H_e = \frac{(3b + 5zd_c)\ d_c}{(2b + 4zd_c)} \qquad (5.4\text{-}20)$$

HYDRAULICS: RELATIONSHIP BETWEEN DEPTH OF FLOW AND SPECIFIC ENERGY FOR RECTANGULAR SECTION

Equations:

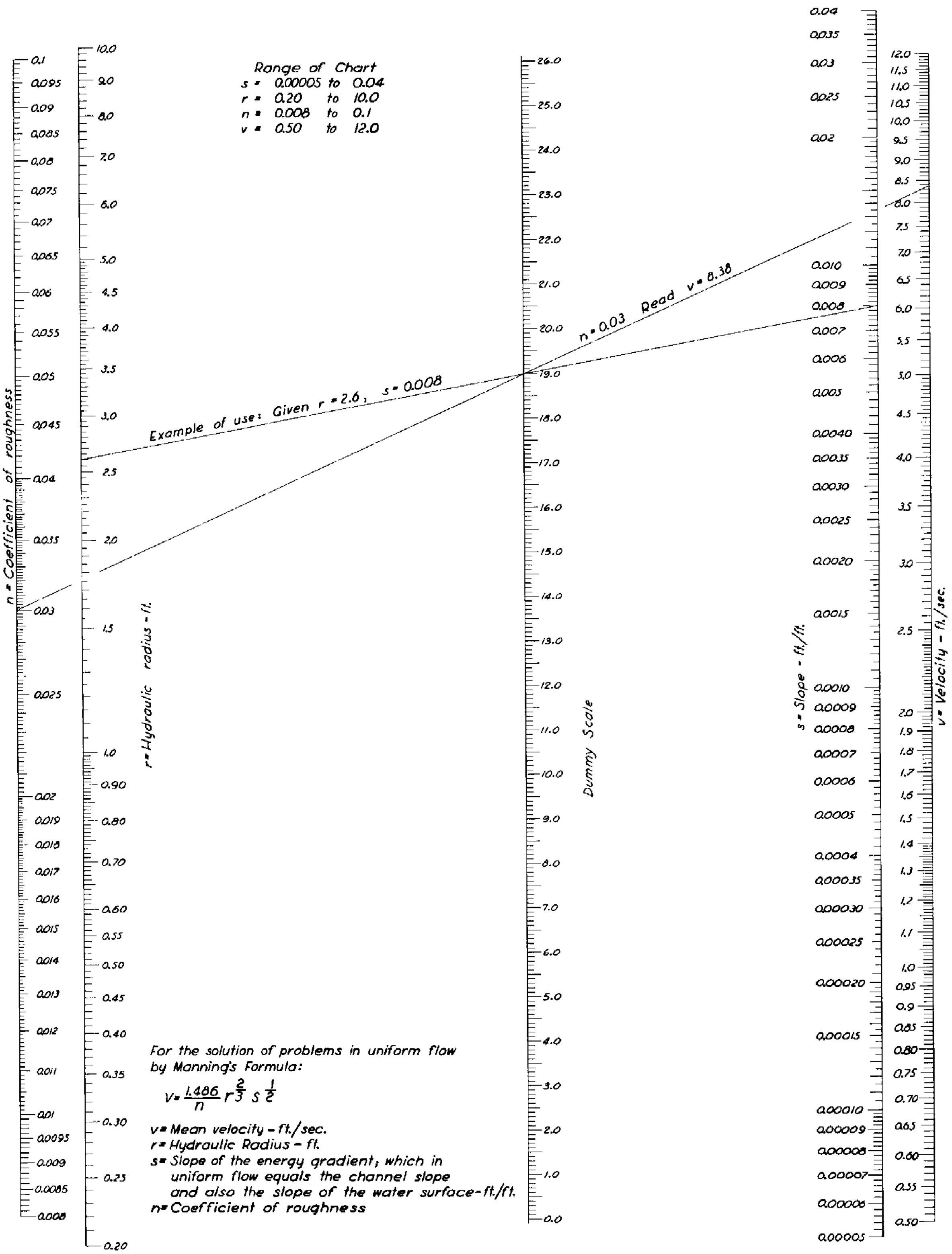
$$H_e = d + \frac{v^2}{2g} = d + \frac{q^2}{2gd^2}$$ where q = discharge per unit width.

$$d_c = \left(\frac{q_c}{\sqrt{g}}\right)^{\frac{2}{3}} = \frac{2}{3} H_{e\,min.}$$ where d_c = critical depth
q_c = critical discharge per unit width
$H_{e\,min.}$ = minimum energy content



REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Bennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES-35 SHEET 1 OF 1 DATE 6-28-50

HYDRAULICS: MANNINGS FORMULA



Range of Chart
s = 0.00005 to 0.04
r = 0.20 to 10.0
n = 0.008 to 0.1
v = 0.50 to 12.0

For the solution of problems in uniform flow by Manning's Formula:

$$V = \frac{1.486}{n} r^{\frac{2}{3}} s^{\frac{1}{2}}$$

v = Mean velocity - ft./sec.
r = Hydraulic Radius - ft.
s = Slope of the energy gradient; which in uniform flow equals the channel slope and also the slope of the water surface - ft./ft.
n = Coefficient of roughness

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Bennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES - 34 SHEET 1 OF 4 DATE 7 - 11 - 50

REVISED 3-31-51

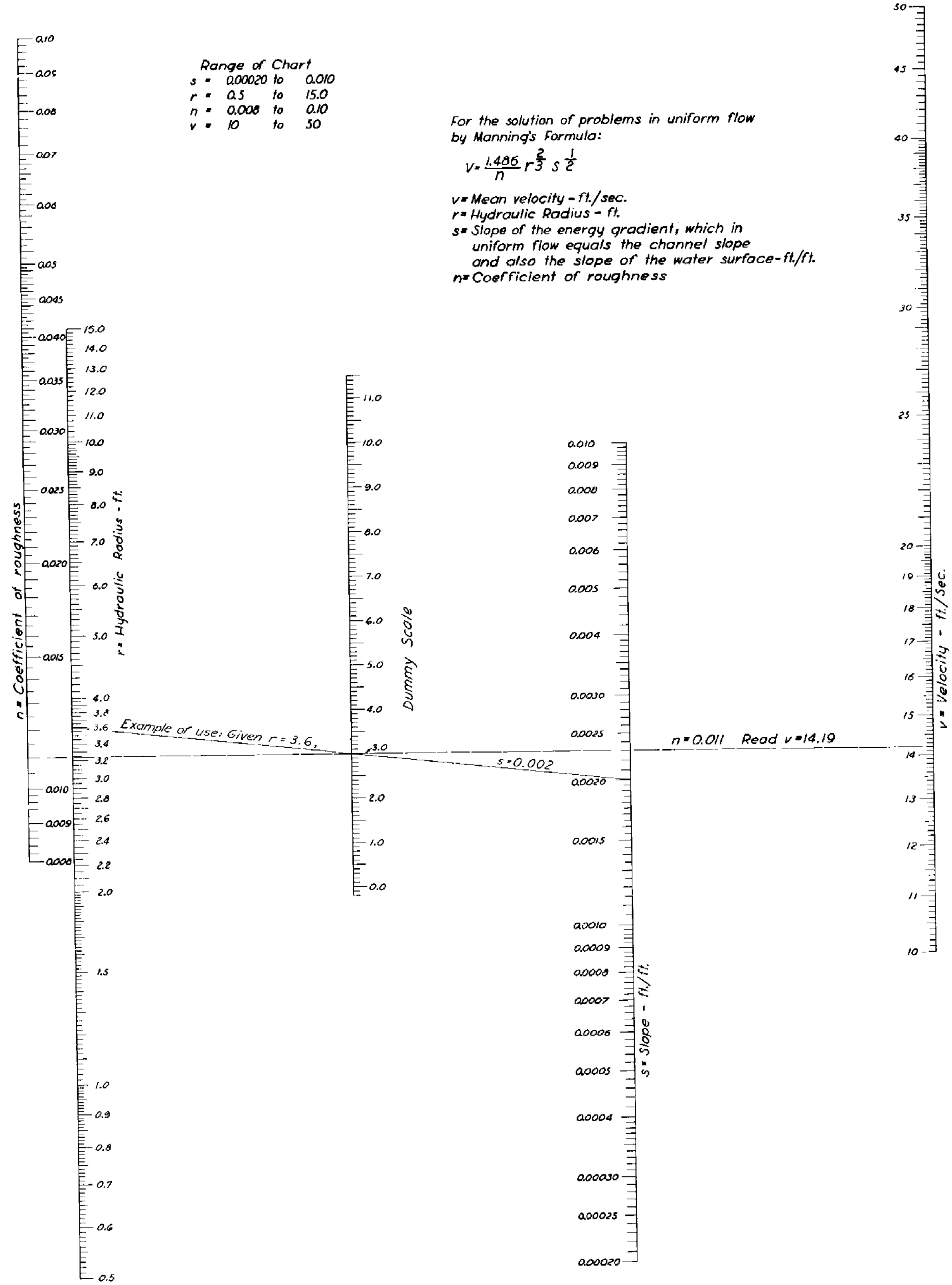
HYDRAULICS: MANNINGS FORMULA

Range of Chart
s = 0.00020 to 0.010
r = 0.5 to 15.0
n = 0.008 to 0.10
v = 10 to 50

For the solution of problems in uniform flow by Manning's Formula:

$$v = \frac{1.486}{n} r^{\frac{2}{3}} s^{\frac{1}{2}}$$

v = Mean velocity - ft./sec.
r = Hydraulic Radius - ft.
s = Slope of the energy gradient, which in uniform flow equals the channel slope and also the slope of the water surface - ft./ft.
n = Coefficient of roughness



REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Bennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES - 34 SHEET 2 OF 4 DATE 7 - 11 - 50

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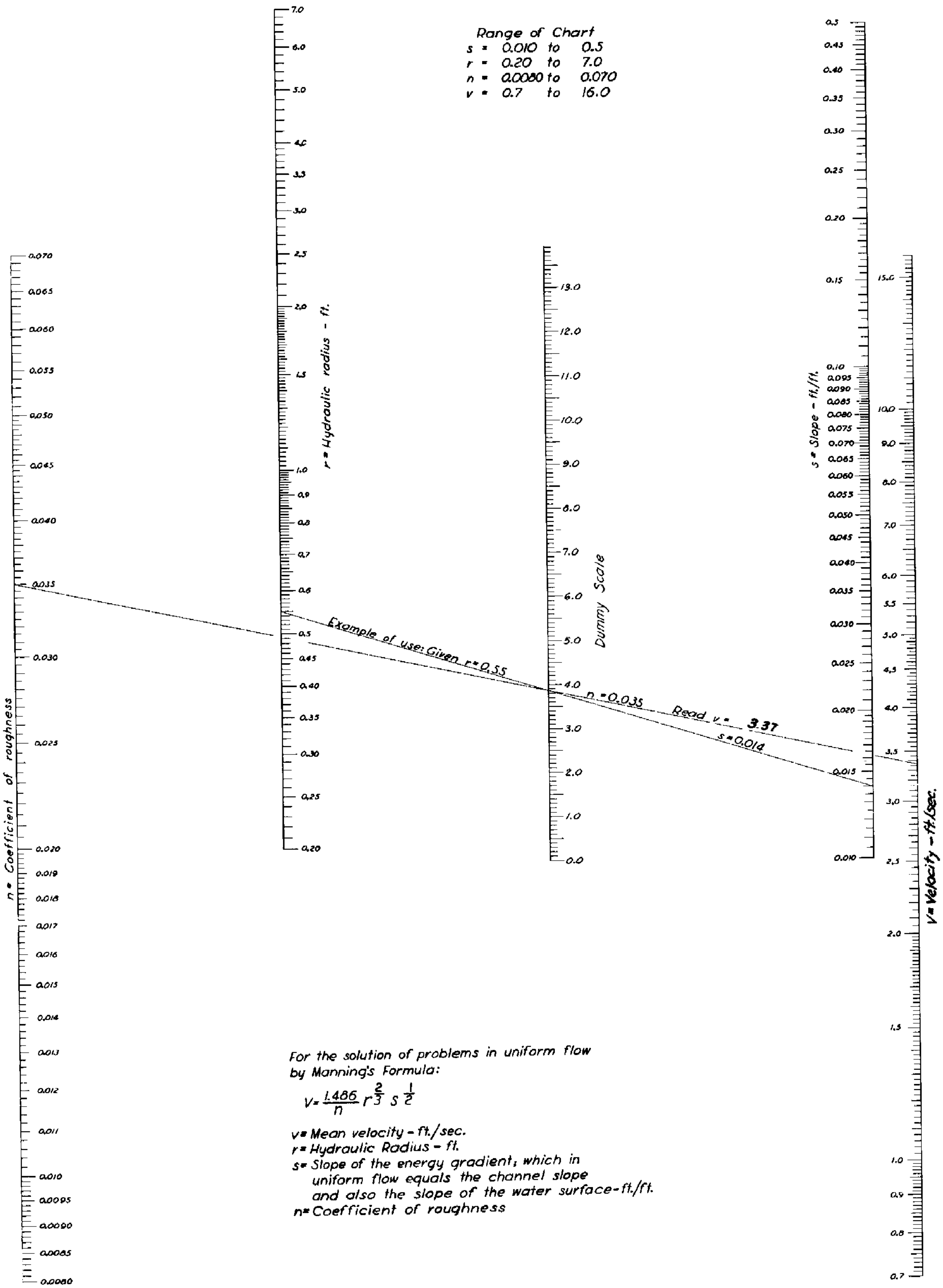
HYDRAULICS: MANNINGS FORMULA



Range of Chart
s = 0.010 to 0.5
r = 0.20 to 7.0
n = 0.0080 to 0.070
v = 0.7 to 16.0

For the solution of problems in uniform flow by Manning's Formula:

$$V = \frac{1.486}{n} r^{\frac{2}{3}} s^{\frac{1}{2}}$$

v = Mean velocity - ft./sec.
r = Hydraulic Radius - ft.
s = Slope of the energy gradient, which in uniform flow equals the channel slope and also the slope of the water surface - ft./ft.
n = Coefficient of roughness

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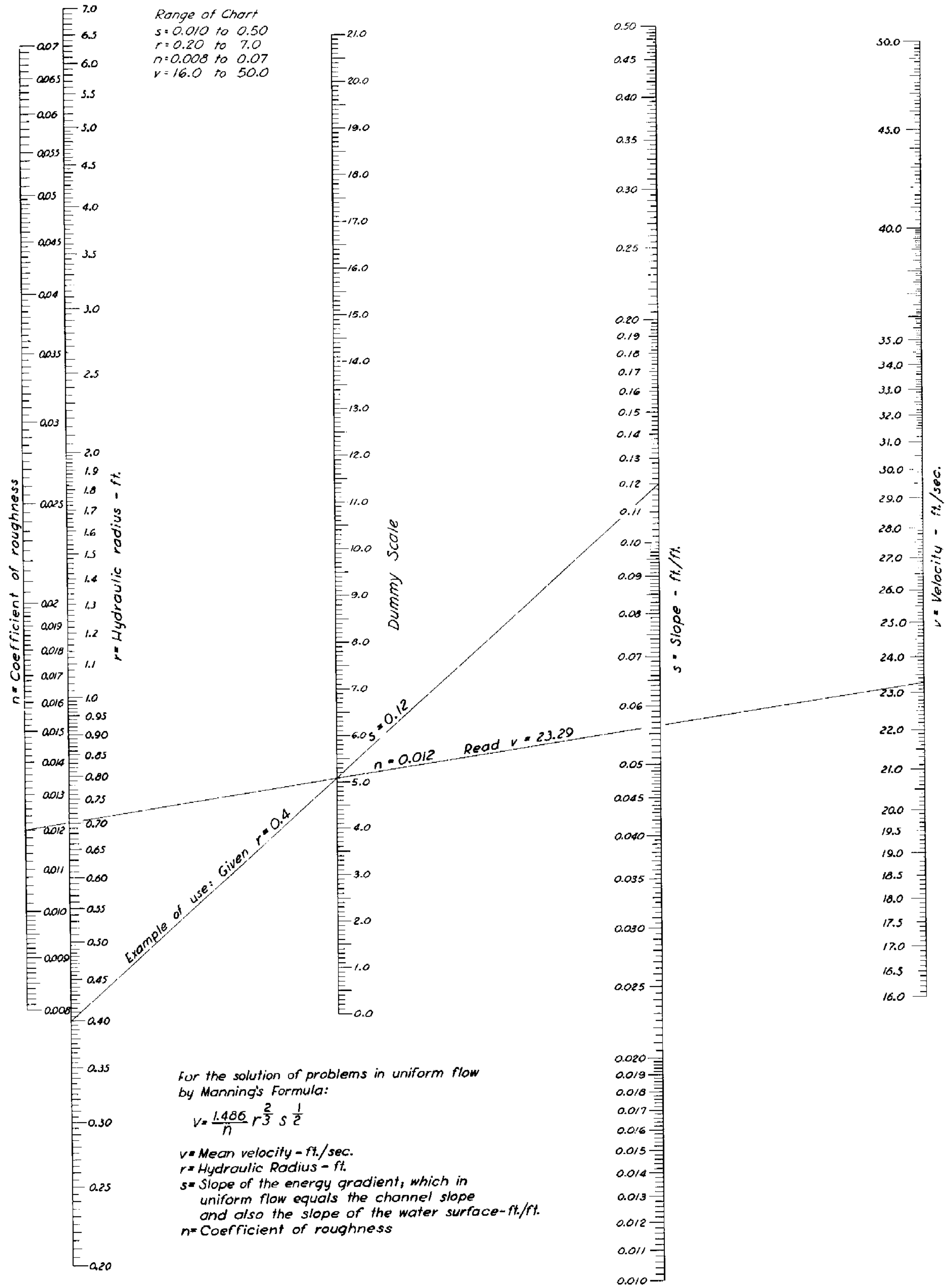
HYDRAULICS: MANNINGS FORMULA



For the solution of problems in uniform flow by Manning's Formula:

$$V = \frac{1.486}{n} r^{\frac{2}{3}} s^{\frac{1}{2}}$$

v = Mean velocity - ft./sec.
r = Hydraulic Radius - ft.
s = Slope of the energy gradient; which in uniform flow equals the channel slope and also the slope of the water surface - ft./ft.
n = Coefficient of roughness

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HYDRAULICS: PRESSURE DIAGRAMS AND METHODS OF COMPUTING HYDROSTATIC LOADS

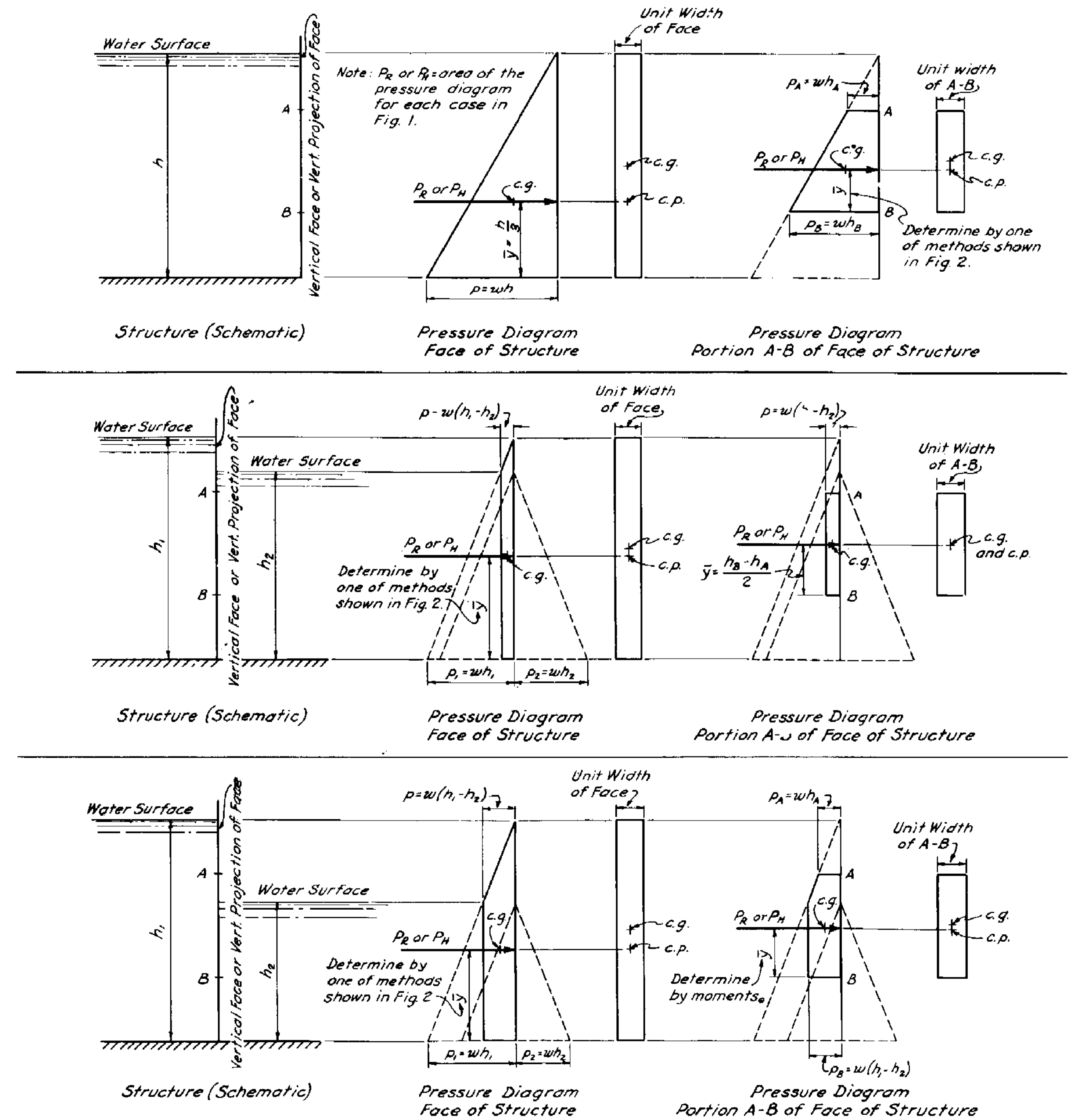

Figure 1

Determination of Center of Gravity of Trapezoid

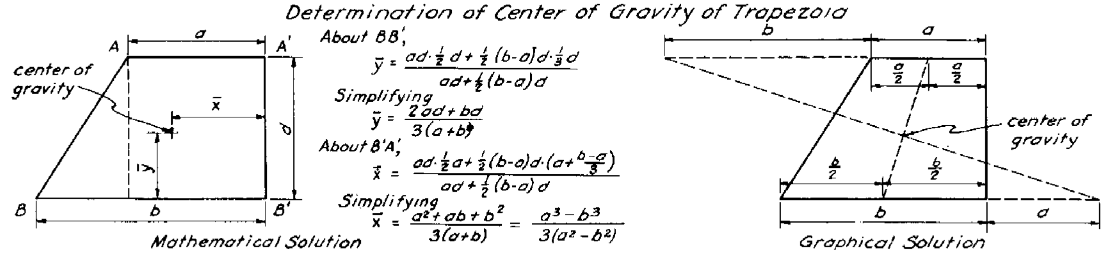

Figure 2

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Bennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES - 31 SHEET 1 OF 2 DATE 6-8-50

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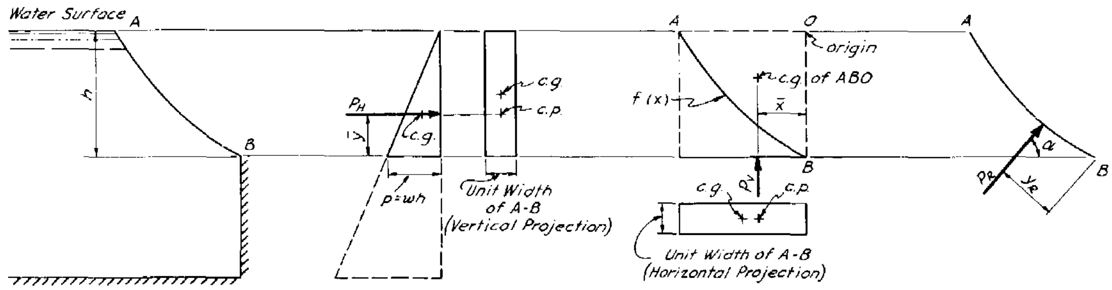
HYDRAULICS: PRESSURE DIAGRAMS AND METHODS OF COMPUTING HYDROSTATIC LOADS



Note: The following solution is general and is valid for, (1), any shape of portion A-B and, (2), water on either side of A-B. However, when water is on the right of A-B, the pressure forces P_H, P_V, and P_R act in directions opposite to those shown.

$$P_H = \frac{1}{2} w h^2 \qquad \bar{y} = \frac{h}{3}$$

$$P_V = w A_{ABO}$$

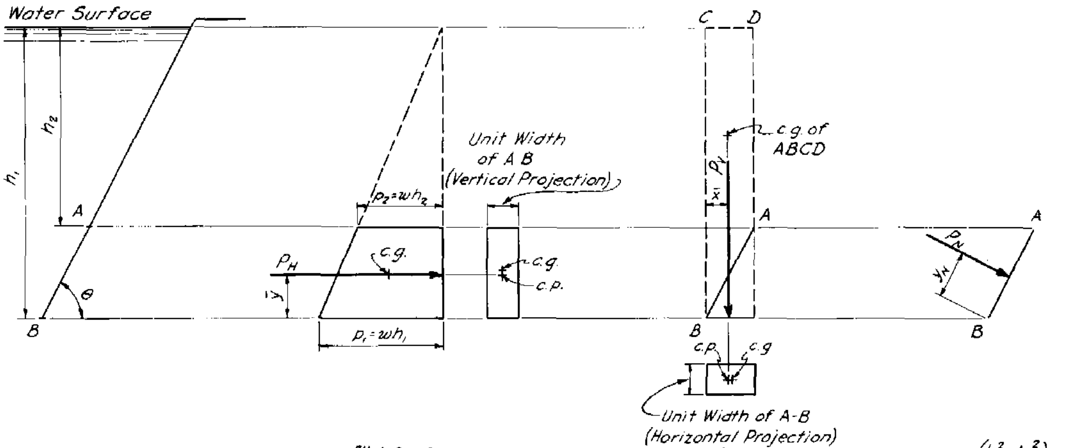
A_{ABO} = Area bounded by ABO $= \int_0^A f(x)\,dx$

$$\bar{x} = \frac{\int_0^A x f(x)\,dx}{A_{ABO}}$$

$$P_R = \sqrt{P_H^2 + P_V^2} \qquad \sin\alpha = \frac{P_V}{P_R} \qquad y_R = \frac{P_H \cdot \frac{h}{3} + P_V \cdot \bar{x}}{P_R}$$

Structure (Schematic) | Pressure Diagram of Horizontal Pressure Portion A-B | Vertical Pressure Force Portion A-B | Resultant Pressure Force Portion A-B

Figure 3



$$P_H = \frac{w}{2}(h_1^2 - h_2^2) \qquad \bar{y} = \frac{(h_1 - h_2)(h_1 + 2h_2)}{3(h_1 + h_2)}$$

$$P_V = \frac{w(h_1^2 - h_2^2)}{2\tan\theta} \qquad \bar{x} = \frac{(h_1 - h_2)(h_1 + 2h_2)}{3(h_1 + h_2)\tan\theta}$$

$$P_N = \frac{w(h_1^2 - h_2^2)}{2\sin\theta} \qquad y_N = \frac{(h_1 - h_2)(h_1 + 2h_2)}{3(h_1 + h_2)\sin\theta}$$

Structure (Schematic) | Pressure Diagram of Horizontal Pressure Portion A-B | Vertical Pressure Force Portion A-B | Normal Pressure Force Portion A-B

Figure 4

Symbols and Definitions

- c.g. — center of gravity of area, as indicated.
- c.p. — center of pressure; i.e., point of action of a pressure force, or a component of a pressure force, on the face, or a projection of the face, of a structure.
- h — height of water above a point, as indicated in "Structure (Schematic)", or as indicated by subscript.
- p — intensity of pressure at a point indicated by subscript, or at bottom of structure or portion of structure if no subscript is used.
- P_H — horizontal component of pressure force per foot width.
- P_V — vertical component of pressure force per foot width.
- P_N — normal pressure force per foot width.
- P_R — resultant pressure force per foot width.
- w — weight of water per cubic foot.
- $\bar{x}, \bar{y}$ — coordinates of c.g. of pressure diagram.
- y_N — distance from given point perpendicular to line of action of P_N.
- y_R — distance from given point perpendicular to line of action of P_R.

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Bennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES - 31 SHEET 2 OF 2 DATE 6-8-50

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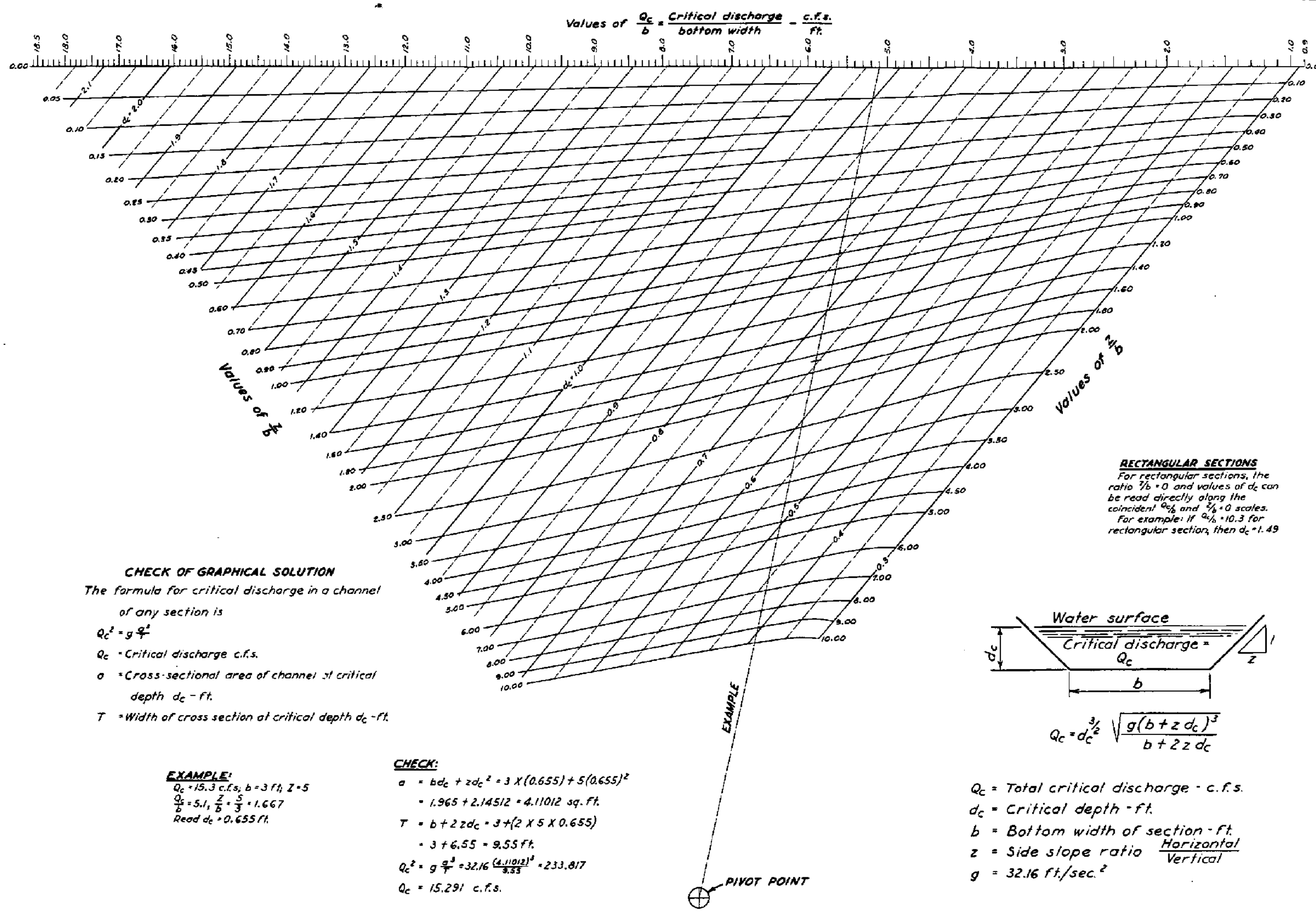
HYDRAULICS: CRITICAL DEPTHS AND DISCHARGES IN TRAPEZOIDAL AND RECTANGULAR SECTIONS



RECTANGULAR SECTIONS

For rectangular sections, the ratio $z/b = 0$ and values of d_c can be read directly along the coincident Q_c/b and $z/b = 0$ scales.

For example: If $Q_c/b = 10.3$ for rectangular section, then $d_c = 1.49$

CHECK OF GRAPHICAL SOLUTION

The formula for critical discharge in a channel of any section is

$Q_c^2 = g \frac{a^3}{T}$

Q_c = Critical discharge c.f.s.

a = Cross-sectional area of channel at critical depth d_c - ft.

T = Width of cross section at critical depth d_c - ft.

$$Q_c = d_c^{3/2} \sqrt{\frac{g(b + z\,d_c)^3}{b + 2z\,d_c}}$$

Q_c = Total critical discharge - c.f.s.

d_c = Critical depth - ft.

b = Bottom width of section - ft.

z = Side slope ratio $\frac{\text{Horizontal}}{\text{Vertical}}$

g = 32.16 ft./sec.^{2}

EXAMPLE:

$Q_c = 15.3$ c.f.s; $b = 3$ ft; $Z = 5$

$\frac{Q_c}{b} = 5.1$, $\frac{Z}{b} = \frac{5}{3} = 1.667$

Read $d_c = 0.655$ ft.

CHECK:

$a = bd_c + zd_c^2 = 3 \times (0.655) + 5(0.655)^2$

$= 1.965 + 2.14512 = 4.11012$ sq. ft.

$T = b + 2zd_c = 3 + (2 \times 5 \times 0.655)$

$= 3 + 6.55 = 9.55$ ft.

$Q_c^2 = g \frac{a^3}{T} = 32.16 \frac{(4.11012)^3}{9.55} = 233.817$

$Q_c = 15.291$ c.f.s.

REFERENCE This nomogram was developed by Paul D. Doubt of the Engineering Standards Unit.

U. S. DEPARTMENT OF AGRICULTURE
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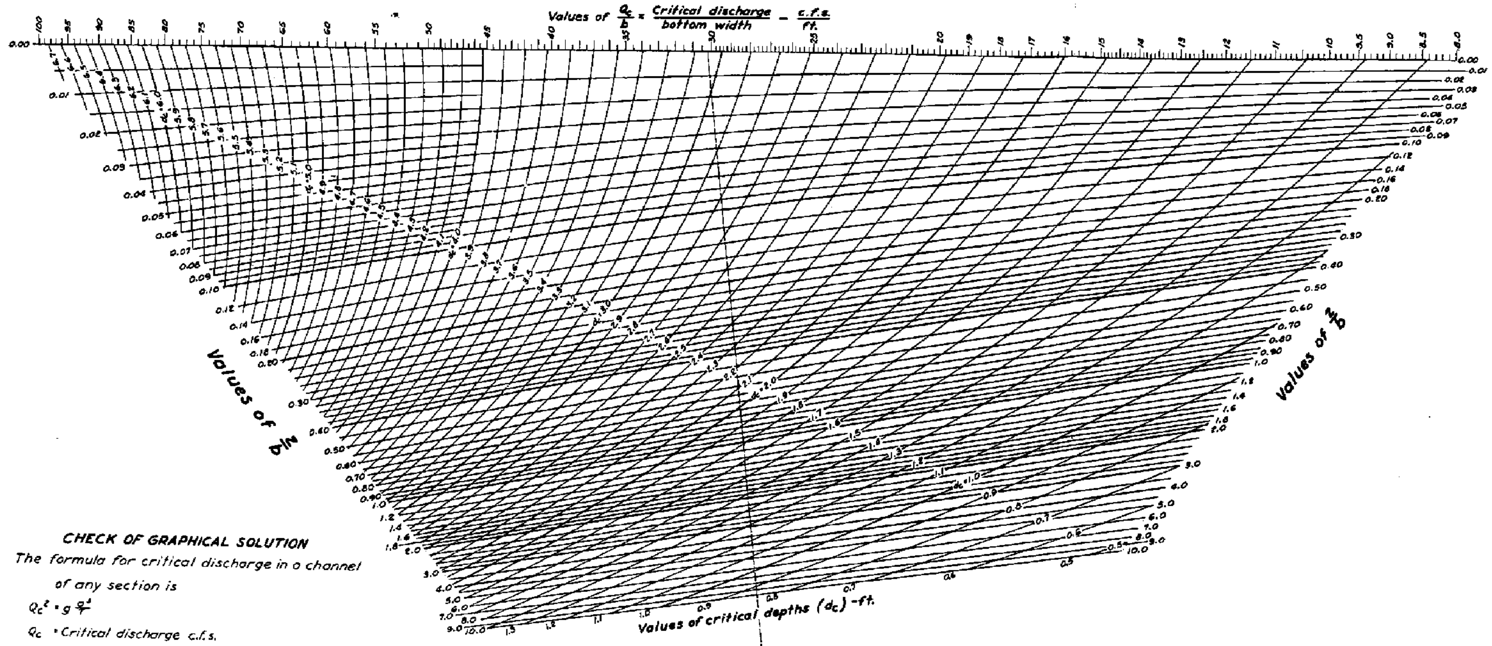
REVISED 3-30-51

HYDRAULICS: CRITICAL DEPTHS AND DISCHARGES IN TRAPEZOIDAL AND RECTANGULAR SECTIONS



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Q_c = Critical discharge c.f.s.

a = Cross-sectional area of channel at critical depth d_c - ft.

T = Width of cross section at critical depth d_c - ft.

EXAMPLE:

$b = 10$ ft; $z = 4$, $Q_c = 302$ cfs

$\frac{Q_c}{b} = 30.2$; $\frac{z}{b} = 0.4$

Read $d_c = 2.26$ ft.

CHECK:

$a = bd_c + zd_c^2 = 10 \times 2.26 + 4(2.26)^2$

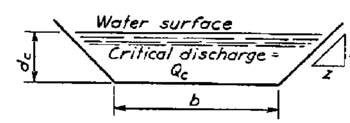
$= 22.6 + 20.43 = 43.03$ sq. ft.

$T = b + 2zd_c = 10 + (2 \times 4 \times 2.26)$

$= 10 + 18.08 = 28.08$ ft.

$Q_c^2 = g\frac{a^3}{T} = 32.16\frac{(43.03)^3}{28.08} = 91,252.5$

$Q_c = \sqrt{91,252.5} = 302.08$



$$Q_c = d_c^{3/2}\sqrt{\frac{g(b + z\,d_c)^3}{b + 2z\,d_c}}$$

Q_c = Total critical discharge - c.f.s.

d_c = Critical depth - ft.

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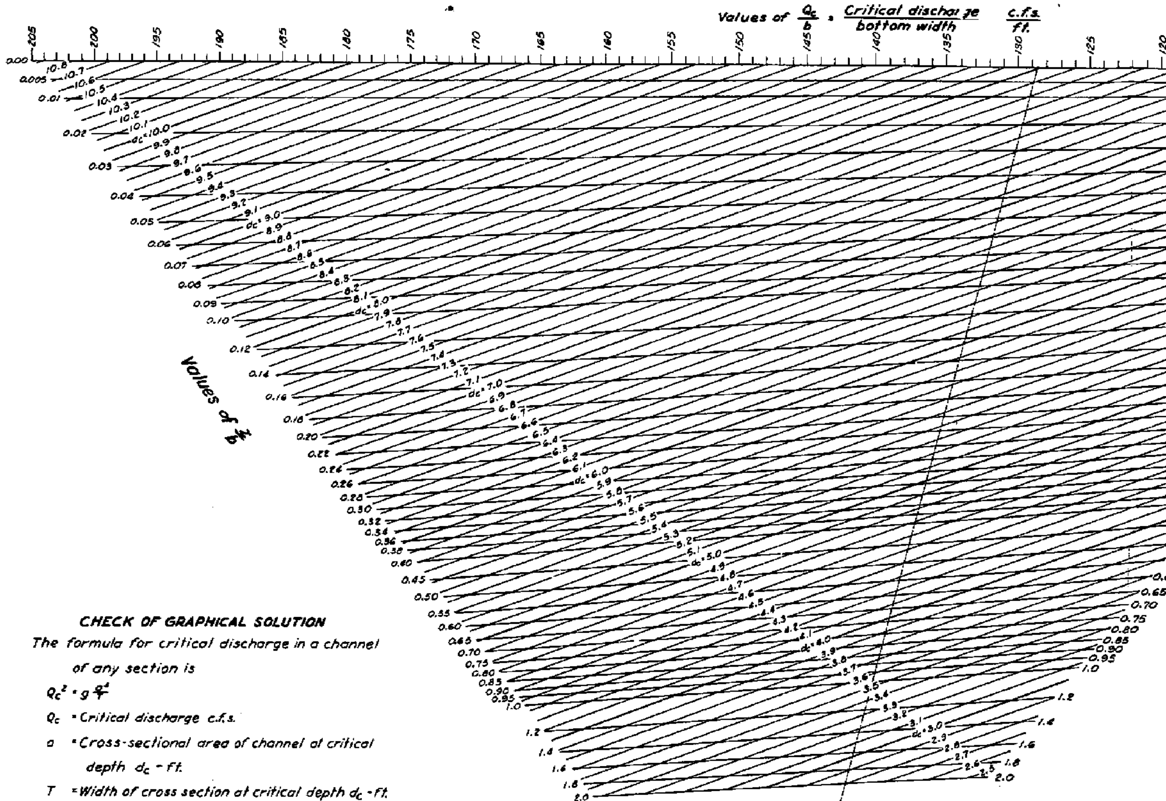
HYDRAULICS: CRITICAL DEPTHS AND DISCHARGES IN TRAPEZOIDAL AND RECTANGULAR SECTIONS



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EXAMPLE:

$Q_c = 3217.5$ cfs; $z = 3.5$; $b = 25$ ft.

$\frac{Q_c}{b} = 128.7$; $\frac{z}{b} = .14$

Read $d_c = 6.04$ ft.

CHECK:

$a = bd_c + zd_c^2 = 25 \times 6.04 + 3.5(6.04)^2$

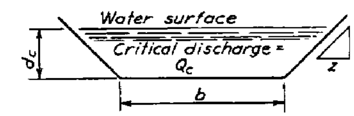
$= 151 + 127.69 = 278.69$ sq. ft.

$T = b + 2zd_c = 25 + (2 \times 3.5 \times 6.04)$

$= 25 + 42.28 = 67.28$ ft.

$Q_c^2 = g\frac{a^3}{T} = 32.16\frac{(278.69)^3}{67.28} = 10,346,518$

$Q_c = \sqrt{10,346,518} = 3,216$ c.f.s.



$$Q_c = d_c^{3/2}\sqrt{\frac{g(b+z\,d_c)^3}{b+2z\,d_c}}$$

Q_c = Total critical discharge - c.f.s.
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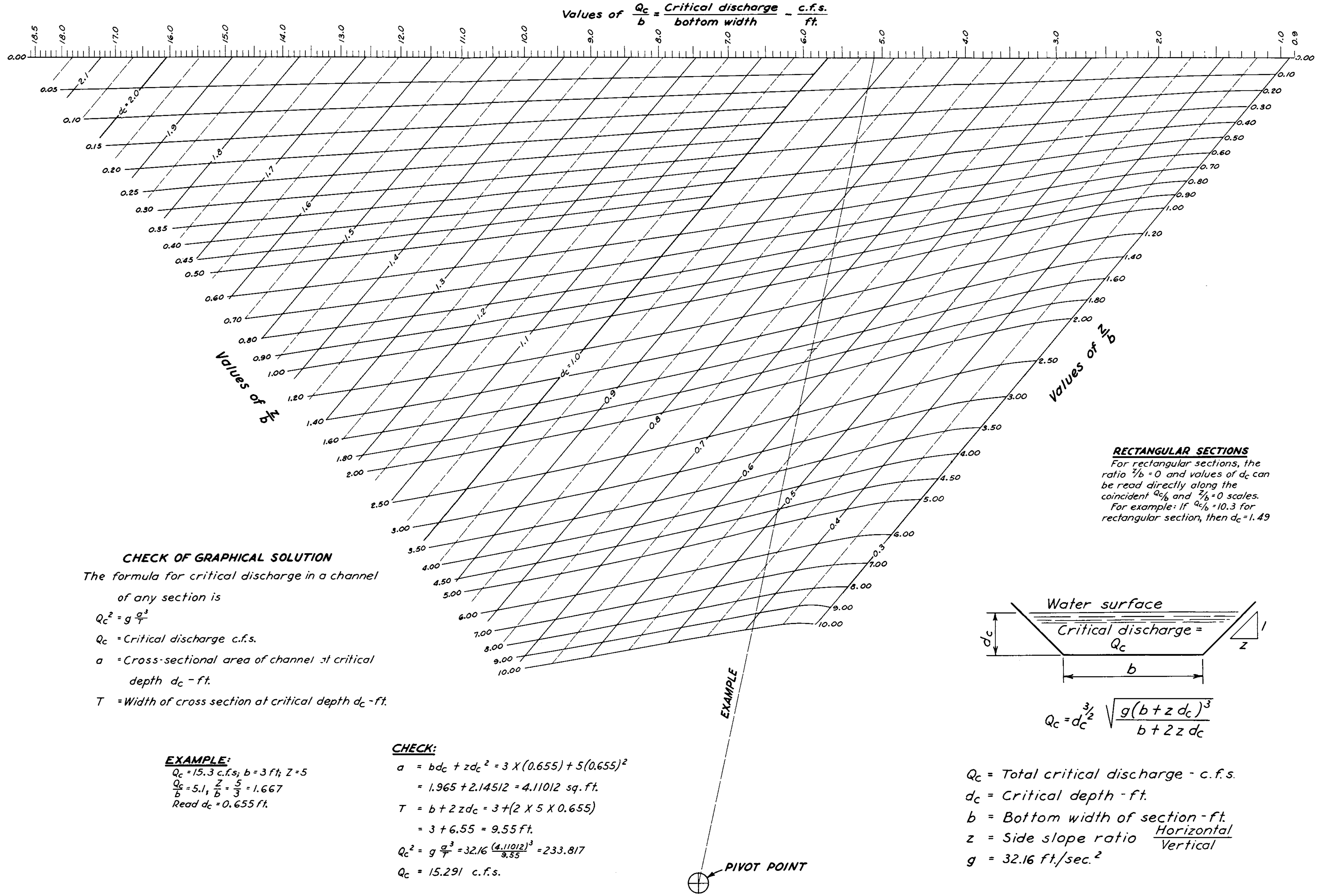
HYDRAULICS: CRITICAL DEPTHS AND DISCHARGES IN TRAPEZOIDAL AND RECTANGULAR SECTIONS



RECTANGULAR SECTIONS

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Water surface
Critical discharge = Q_c
d_c
b
1
z

$$Q_c = d_c^{3/2} \sqrt{\frac{g(b + z\,d_c)^3}{b + 2z\,d_c}}$$

Q_c = Total critical discharge - c.f.s.
d_c = Critical depth - ft.
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z = Side slope ratio $\frac{\text{Horizontal}}{\text{Vertical}}$
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$Q_c = 15.3$ c.f.s.; $b = 3$ ft; $Z = 5$
$\frac{Q_c}{b} = 5.1$, $\frac{Z}{b} = \frac{5}{3} = 1.667$
Read $d_c = 0.655$ ft.

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U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Bennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES - 24 SHEET 1 OF 3 DATE 5-2-50

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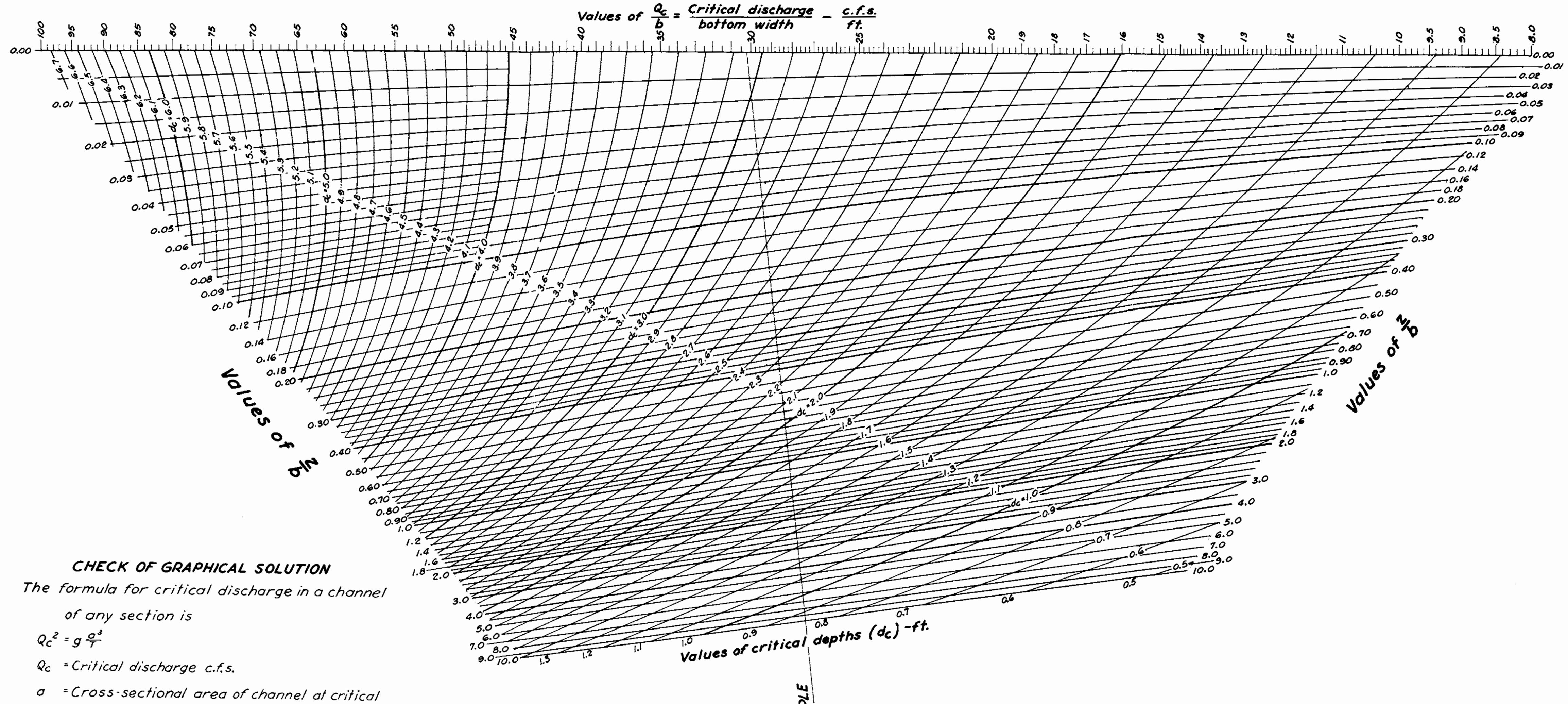
HYDRAULICS: CRITICAL DEPTHS AND DISCHARGES IN TRAPEZOIDAL AND RECTANGULAR SECTIONS



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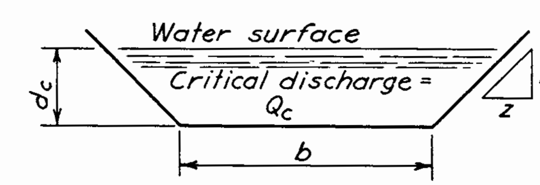

$$Q_c = d_c^{3/2}\sqrt{\frac{g(b+z\,d_c)^3}{b+2z\,d_c}}$$

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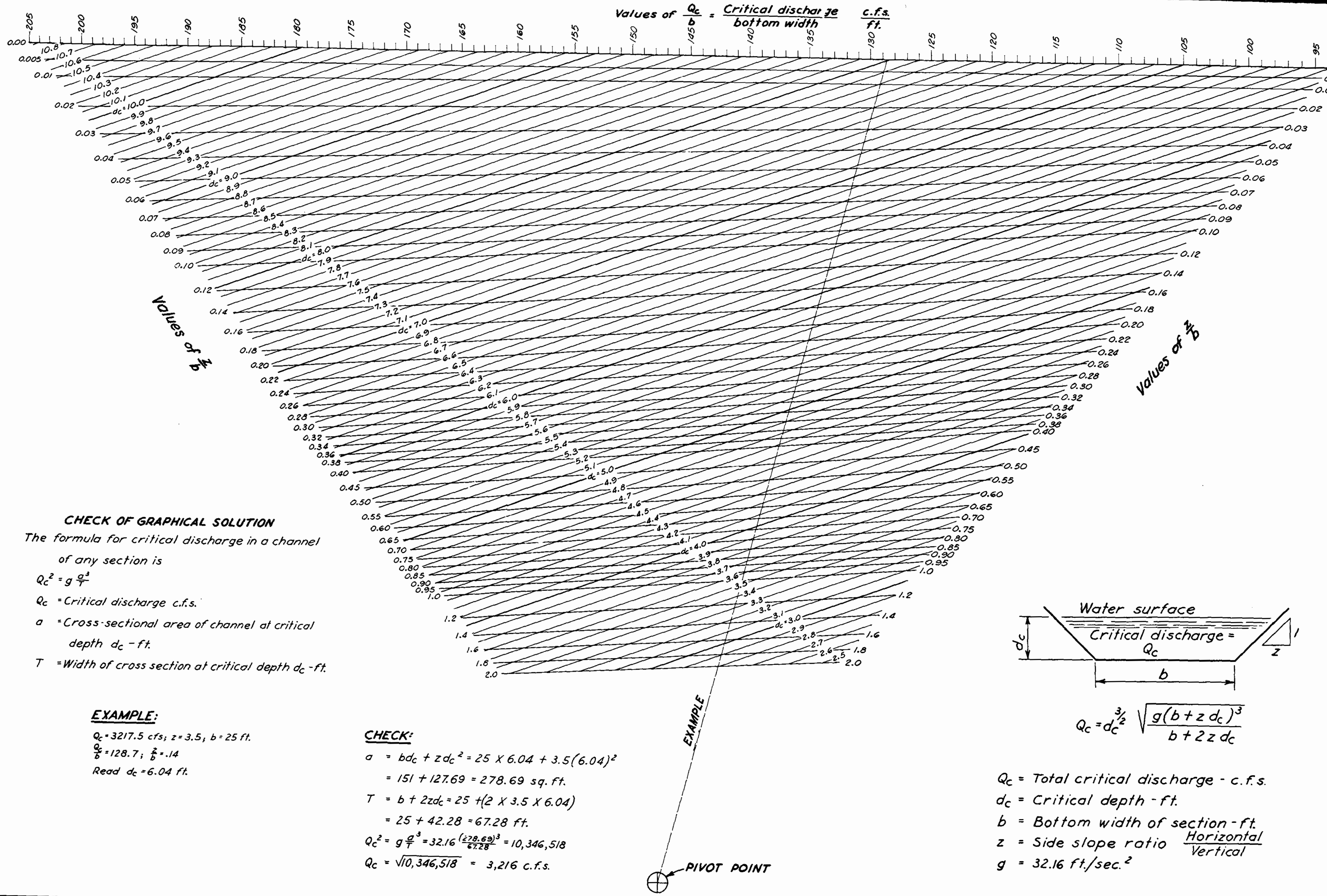
HYDRAULICS: CRITICAL DEPTHS AND DISCHARGES IN TRAPEZOIDAL AND RECTANGULAR SECTIONS



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Read $d_c = 6.04$ ft.

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$$Q_c = d_c^{3/2}\sqrt{\frac{g(b+zd_c)^3}{b+2zd_c}}$$

Q_c = Total critical discharge - c.f.s.

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$$d_c = \frac{v_c^2}{g} - \frac{b}{2z} + \sqrt{\frac{b^2}{4z^2} + \frac{v_c^4}{g^2}} \quad (5.4\text{-}21)$$

$$v_c = \sqrt{\left(\frac{b + zd_c}{b + 2zd_c}\right) d_c g} \quad (5.4\text{-}22)$$

$$Q_c = d_c^{3/2} \sqrt{\frac{g(b + zd_c)^3}{b + 2zd_c}} \quad (5.4\text{-}23)$$

The use of these formulas will be materially simplified by tables referred to in "King's Handbook", pp. 382-384. Drawing ES-24 is an alignment chart to be used in solving for Q_c or d_c in equation (5.4-23).

4.5.4 Critical Slope. Critical slope is that slope which will sustain a given discharge in a given channel at uniform, critical depth. The relationships that must exist between discharge, energy, and depth in critical flow are expressed by equations (5.4-3) and (5.4-4). The slope, roughness coefficient, and shape of channel cross section determine whether flow will occur in accordance with these specific relationships. A channel of given cross section, slope, and roughness coefficient will carry only one discharge at uniform, critical depth; the uniform depths at which other discharges will occur are either greater or less than the critical depths for the respective discharges. The same fact stated in another manner is: A channel having a given cross section, roughness coefficient, and discharge will carry that discharge at uniform, critical depth if the channel slope is equal to the critical slope; if the channel slope is greater than the critical slope, the depth of flow will be less than critical; if the channel slope is less than critical, the depth of flow will be greater than critical.

From the critical flow formulas and Manning's formula,

$$v_c = \sqrt{gd_m} \quad \text{and}$$

$$v = \frac{1.486}{n} r^{2/3} s^{1/2}, \text{ then}$$

$$\frac{1.486}{n} r^{2/3} s_c^{1/2} = \sqrt{gd_m}$$

From which the critical slope, s_c, is:

$$s_c = 14.56 \frac{n^2 d_m}{r^{4/3}} \quad (5.4\text{-}24)$$

In a given channel the following criteria apply:

If the channel slope = $14.56\ n^2 d_m / r^{4/3}$, depth of flow = d_c.

If the channel slope $< 14.56\ n^2 d_m / r^{4/3}$, depth of flow $> d_c$.

If the channel slope $> 14.56\ n^2 d_m / r^{4/3}$, depth of flow $< d_c$.

Formula (5.4-24) and the above criteria are useful for the following purposes: locating control sections, guiding the selection of channel section and grade in preliminary design so that the unstable conditions of uniform flow at critical depth may be avoided, determining the type of water surface curve that will occur in a given reach of channel.

4.5.5 Significance of Critical Flow in Design. Critical, subcritical and supercritical flow affect design in the following manner:

(a) Critical flow. Uniform flow at or near critical depth is unstable. This results from the fact that the unique relationship between energy head and depth of flow which must exist in critical flow is readily disturbed by minor changes in energy. Examine the curve for q = 40 c.f.s. on drawing ES-35. The critical depth is 3.68 feet and the corresponding energy head is 5.52. If the energy head is increased to 5.60, the depth may be 3.2 or 4.2. Those who have seen uniform flow at or near critical depth have observed the unstable, wavy surface that is caused by appreciable changes in depth resulting from minor changes in energy. In channel design these conditions must be recognized. Variations in channel roughness, cross section, slope, or minor deposits of sediment or debris may cause fluctuations in depth of flow that are significant to channel operation. In many cases it is desirable to base design computations on two or more values of n in order to establish the probable range of operating conditions. Because of the unstable flow, channels carrying uniform flow at or near critical depth should not be used unless the situation allows no alternative.

The critical flow principle is the basis for the design of control sections at which a definite stage-discharge relation is desired or required.

(b) Subcritical flow. Two general characteristics of subcritical flow are important. First, at all stages in the subcritical range, except those in the immediate vicinity of the critical, the velocity head is small in comparison with the depth of flow. Study of the curves of constant discharge, drawing ES-35, will make this point clear. Second, the velocities are less than wave velocity for the depths involved and a backwater curve will result from retardation of velocity. Thus, in the subcritical range we are concerned with cases in which the depth of flow is of greater importance than kinetic energy as represented by velocity head. In practice, this means that changes in channel cross section, slope, roughness, and alignment may be made without the danger of developing seriously disturbed flow conditions so long as the design assures that flow in the supercritical range will not be created for some discharges in the operational range.

However, in many cases the latitude in design which may be possible as a result of dealing with subcritical flow will be offset by limited head requiring that friction losses be held to a minimum.

(c) Supercritical flow. The design of structures to carry supercritical flow requires consideration of some of the most complex problems in hydraulics. In supercritical flow the velocity head may range from a value approximately equal to depth of flow to many times the depth of flow. Note from the curves on drawing ES-35 that the velocity head increases very rapidly with decreases in depth throughout the supercritical range. Supercritical velocities exceed the velocities at which gravity waves may be propagated upstream. Any obstruction of flow will result in a standing wave, and there will be no effect upon flow upstream from the obstruction. The fact that kinetic energy predominates in supercritical flow and cannot be dissipated through the development of a water surface curve extending upstream is of primary importance in design.

Channels involving changes of direction, contraction or expansion of cross section, or the joining of two flows at a confluence at which either or both of the flows may be supercritical require careful consideration. Changes in direction or channel contractions develop disturbances at the walls of the channel which take the form of standing waves reflected diagonally from wall to wall downstream from the disturbance points. The height of these standing waves may be several times the depth of flow immediately upstream from the origin of the disturbance. Confluences at which either flow or both flows may be supercritical also develop disturbances resulting in standing waves. In expanding channels the discharge may be incapable of following the channel walls because of the high velocities involved. This results in nonuniform depth and the development of a hydraulic jump which is unstable as to both location and height.

A number of the factors that must be determined as a basis for design of these high velocity structures cannot be evaluated through theoretical analyses only. General experimental results as well as experimentation with individual structures are required. Basic requirements for projects and structures should first be determined and tentative designs to meet these requirements selected. The tentative designs should then be perfected through model tests.

Water surface profiles applying to cases of supercritical flow in straight channels of uniform width can normally be determined with sufficient accuracy for design by standard methods of analysis. Most structures must have outlet velocities in the subcritical range to prevent erosion damage. The creation of the hydraulic jump by the use of stilling basins is an efficient means of dissipating the excessive energy in supercritical flow. Design criteria, based on thorough model investigation, are available for some types of stilling basins. An example is the SAF stilling basin. Under unusual conditions or when exacting requirements must be met, stilling basin designs should also be perfected by model tests.

4.6 The Hydraulic Jump. When water flowing at greater than critical velocity enters water with less than critical velocity and sufficient depth, a hydraulic jump develops. In the jump the depth increases from an original depth to a depth which is less than the higher of the two alternate depths of equal energy. The depth before the jump is always less than critical, and the depth after the jump is greater than critical.

Between cross sections located just upstream and downstream from the jump there occurs a loss of energy, a decrease in velocity, and an increase in hydrostatic pressure. "King's Handbook", pp. 373-378 and 406-412, gives a discussion of the energy and momentum conditions involved in the hydraulic jump and shows the derivation of general formulas.

In paragraph 4.5 the specific energy in flow is discussed and illustrated by the curves on drawing ES-35. The force of a flowing stream is the momentum force due to velocity plus the hydrostatic pressure force. The force equation is:

$$F_m = \frac{Q^2}{ga} + a\bar{y} \qquad (5.4\text{-}25)$$

F_m = the force of the stream.
Q = the discharge.
a = the cross-sectional area.
$\bar{y}$ = the depth to the center of gravity of the cross section.
g = the acceleration of gravity.

For a rectangular channel of unit width, equation (5.4-25) becomes:

$$F_m = \frac{q^2}{gd} + \frac{d^2}{2}$$

Drawing ES-36 shows the specific energy curve, the momentum force curve, and a sketch of a hydraulic jump for a discharge of 30 c.f.s. in a rectangular channel of unit width. The momentum force curve for any discharge in any type of channel would be similar to the one shown. Note that there is a depth at which the force of the flowing stream is minimum and this depth is the critical depth. When the force is greater than the minimum, there are two possible depths, called conjugate or sequent depths, of flow. One of these depths is less than critical, that is, in the supercritical range where the pressure force, because of shallow depth, is relatively low and the momentum force, because of high velocity, is relatively great. The other depth, the sequent depth, is in the subcritical range where the pressure force becomes more significant than the momentum force. The lesser of the two depths is the depth before a jump and the greater is the depth after a jump. The energy head lost in the jump is the difference between the energy heads for these two depths. As the two depths of equal force approach the critical depth, the energy loss in the jump decreases.

4.6.1 Depth After the Jump. Formulas from which depth before and after the jump in any type channel may be computed are:

HYDRAULICS: LOSS IN ENERGY HEAD DUE TO HYDRAULIC JUMP IN RECTANGULAR CHANNEL

Equations and Symbols:

$H_e = d + \frac{q^2}{2gd^2}$ = specific energy

$F_m = \frac{q^2}{gd} + \frac{d^2}{2}$ = momentum force

d = depth of flow.

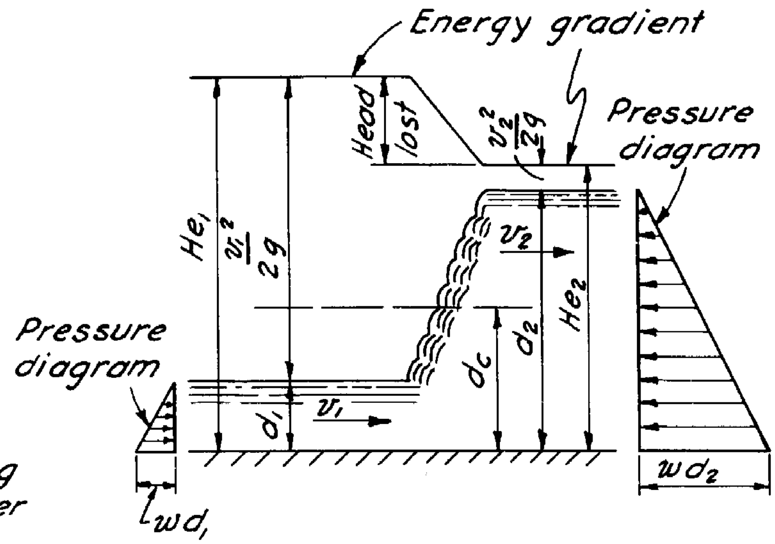
d_c = critical depth.

v = velocity of flow.

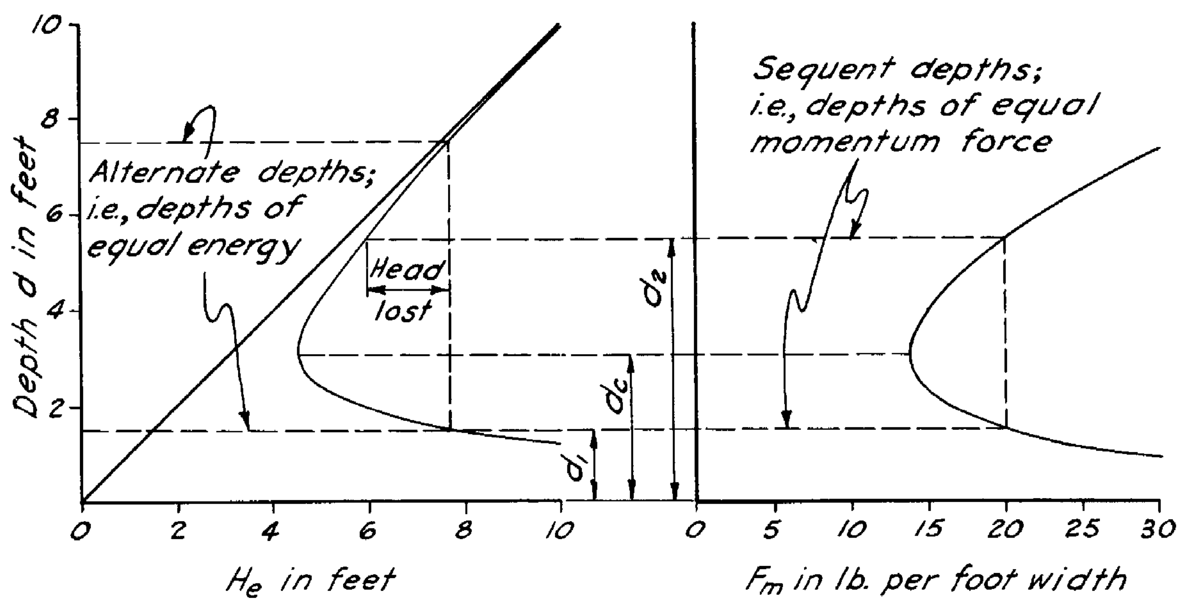
q = discharge per foot width.

w = weight of water per cubic foot.

1 & 2 = subscripts denoting section before and after jump respectively.



Hydraulic Jump



Specific Energy Diagram for q = 30 c.f.s. per foot width

Momentum Force Diagram for q = 30 c.f.s per foot width

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$$v_1^2 = g\left[\frac{a_2\bar{y}_2 - a_1\bar{y}_1}{a_1\left(1 - \frac{a_1}{a_2}\right)}\right] \tag{5.4-26}$$

$$Q^2 = g\left(\frac{a_2\bar{y}_2 - a_1\bar{y}_1}{\frac{1}{a_1} - \frac{1}{a_2}}\right) \tag{5.4-27}$$

Depths before and after the jump in rectangular sections are given by:

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{2v_1^2 d_1}{g} + \frac{d_1^2}{4}} \tag{5.4-28}$$

$$d_1 = -\frac{d_2}{2} + \sqrt{\frac{2v_2^2 d_2}{g} + \frac{d_2^2}{4}} \tag{5.4-29}$$

Q = discharge.
v = mean velocity.
a = cross-sectional area of flow.
d = depth of flow.
$\bar{y}$ = depth to the center of gravity of the cross section of flow.
g = acceleration of gravity.
Subscripts 1 and 2 denote cross sections and depths before and after the jump respectively.

"King's Handbook", table 133, gives a limited number of values of depth after the jump in rectangular channels, and "Hydraulic Tables", table 3, gives a more complete series of these values. Values for $\bar{y}$ for trapezoidal and circular channels for use in equations (5.4-26) and (5.4-27) can be computed more readily by the use of "King's Handbook", tables 99 and 104.

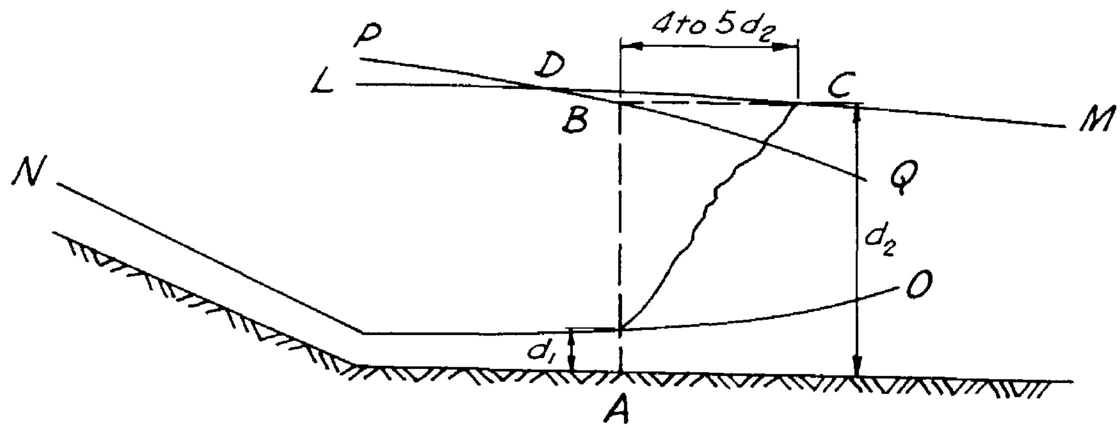
4.6.2 Location of the Jump. When structures involve the hydraulic jump, design will normally be made by criteria that will place the jump in a certain position and no specific estimate of jump location is necessary. However, there will be cases in which a determination of where the jump will occur will be valuable or required. The reliability of the determination depends on the accuracy with which friction loss can be estimated. The location of the jump is estimated by the following steps (see Fig. 5.4-1):

a. From a control section downstream from the jump compute and plot the water surface profile, LM, to a point upstream from the probable position of the jump. And from a control section upstream from the jump compute and plot the water surface profile, NO, to a point downstream from the jump. Methods of computing these profiles are given in paragraphs 4.7.4 and 4.7.5.

b. Through depths sequent to the depths of profile, NO, at 4 to 6 points along a reach certain to include the jump, draw the curve PQ. See paragraph 4.6.1 for methods of determining sequent depths.

c. The approximate location of the jump is D, the intersection of the sequent depth curve, PQ, and the tailwater curve, LM.

d. In most cases the approximate location of the jump, as determined by carrying out steps a, b, and c, will satisfy practical requirements. Some authorities suggest that a closer approximation of the jump location may be obtained as follows: Construct AB vertical and equal in length to the depth d_2 with the jump at A, and BC horizontal and equal in length to the length of jump which may be taken as 4 to 5 d_2. This construction must be in accordance with the horizontal and vertical scales to which the water surface profiles and the sequent depth curve are plotted. Note that the position of point C must meet three simultaneous requirements: First, it is on the tailwater profile; second, the depth d_2 at C is that which is sequent to d_1 at A; third, it is the length of the jump downstream from A.



Location of the Hydraulic Jump

FIG. 5.4-1

4.7 Water Surface Profiles. The main objective in the majority of open channel problems is to determine the profile of the water surface. Methods of computing water surface profiles are described for the general cases of uniform flow, accelerated flow, and retarded flow.

4.7.1 Uniform Flow. In uniform flow the force of gravity is balanced by the friction force. The slopes of the hydraulic gradient, the energy gradient, and the bottom of the channel are equal; also mean velocity, depth of flow, and area of flow are constant from section to section. Depth of uniform flow is called the normal depth. Flow may be uniform only when the channel is uniform in cross section. The relationships between the energy gradient, hydraulic gradient, and bottom of channel are shown by fig. 5.4-2.

The energy equation for sections 1 and 2 is:

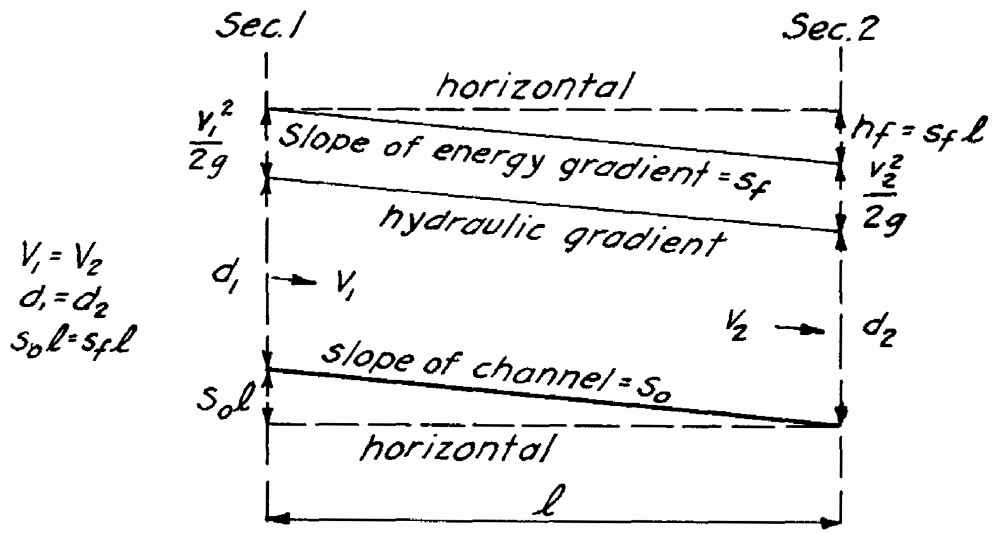
$$\frac{v_1^2}{2g} + d_1 + s_o\ell = \frac{v_2^2}{2g} + d_2 + h_f$$

Since the velocities and depths at sections 1 and 2, and channel slope and friction slope are equal, the friction head, h_f, expressed as a function of velocity, cross section elements, slope, and roughness coefficient, is the basis for the solution of uniform flow problems. Solutions may be made through the use of Manning's formula:

$$v = \frac{1.486}{n} r^{2/3} s^{1/2} \qquad (5.4\text{-}1)$$

$$Q = \frac{1.486}{n} ar^{2/3} s^{1/2} \qquad (5.4\text{-}30)$$

v = mean velocity in ft. per sec.
Q = discharge in cu. ft. per sec.
a = cross-sectional area of flow in sq. ft.
r = hydraulic radius in ft.
s = slope of the energy gradient or friction head in ft. per ft.
n = roughness coefficient.



Conditions of Uniform Flow

FIG. 5.4-2

"King's Handbook", pp. 279-283, gives a number of useful working forms of Manning's formula. The most frequently used of these forms are:

$$Q = \frac{K}{n} d^{8/3} s^{1/2} \tag{5.4-31}$$

$$Q = \frac{K'}{n} (b \text{ or } T)^{8/3} s^{1/2} \tag{5.4-32}$$

$$s = \frac{n^2 v^2}{2.2082 r^{4/3}} \tag{5.4-33}$$

$$h_f = \frac{\ell n^2 v^2}{2.2082 r^{4/3}} \tag{5.4-34}$$

K and K' = factors varying with the ratios of certain linear dimensions of cross sections.
b = bottom width of rectangular or trapezoidal sections.
d = depth of flow in any section.
h_f = total friction head lost in a reach.
T = width of water surface in parabolic sections.
ℓ = horizontal length of reach.

"King's Handbook" contains tables of values of K, K' and $1 \div (2.2082 r^{4/3})$. The utility of formulas (5.4-31) and (5.4-32) depends upon the availability of tables giving values of K and K', and the user should refer to King's Handbook for further discussion regarding application of these formulas to channel sections of different forms. Formulas (5.4-1), (5.4-30), (5.4-33), and (5.4-34) are useful in many problems and they are adapted to use with slide rule or longhand computations. It should be noted that King treats the triangular section as a special form of the trapezoidal section; therefore, K and other factors related to triangular sections are found in the tables for trapezoidal sections where the ratio of d/b = infinity. The alignment chart, drawing ES-34, may be used for graphical solutions for any one unknown in formula (5.4-1).

4.7.2 Accelerated and Retarded Flow. This subsection considers steady, nonuniform flow and attention is given to methods of determining water surface profiles under various conditions. Those who are interested in a more thorough treatment of nonuniform flow are referred to "Steady Flow in Open Channels" by Sherman M. Woodward and Chesley J. Posey, John Wiley and Sons, Inc.; and "Hydraulics of Open Channels" by Boris A. Bakhmeteff, McGraw-Hill Book Company.

In accelerated or retarded flow, as in uniform flow, the fall of the energy gradient represents the loss of head by friction. The fall in the water surface reflects both friction loss and the conversions between potential and kinetic energy. In analyzing nonuniform flow problems it is, therefore, necessary to consider both the hydraulic gradient and the energy gradient.

Refer to fig. 5.4-2. The equation of energy for sections 1 and 2 is:

$$\frac{v_1^2}{2g} + d_1 + s_o \ell = \frac{v_2^2}{2g} + d_2 + s_f \ell \qquad (5.4\text{-}35)$$

Solving for ℓ gives:

$$\ell = \frac{\left(\frac{v_2^2}{2g} + d_2\right) - \left(\frac{v_1^2}{2g} + d_1\right)}{s_o - s_f} \qquad (5.4\text{-}36)$$

v = mean velocity.
d = depth of flow.
s_o = slope of channel.
$s_f = h_f/\ell$ = friction slope, i.e., the slope of the energy gradient.
ℓ = length of reach.
Subscripts 1 and 2 denote upstream and downstream sections respectively.

Water surface profiles may be computed by the use of formula (5.4-35) or (5.4-36). The methods of use of both formulas are among the several step methods for backwater computations. In those cases where the water surface elevation, i.e., the depth, at a specific section is to be computed, the solution is by trial and error and formula (5.4-35) should be used. This method of solution can be applied in any type of channel regardless of whether it is uniform or nonuniform. Where the distances between a series of depths along a uniform channel are to be computed, the solution is direct with formula (5.4-36), but it should be noted that this approach may be used only in uniform channels.

The general procedure for computing depths at given locations is: (1) Determine the location of a control section and the depth of flow at that section. (2) Take a relatively short reach of selected length and assume the depth at the upper or lower end depending on whether the computations are to proceed upstream or downstream. (3) Evaluate s_f and the velocity heads and substitute the values in formula (5.4-35). If the equation balances, the assumed depth is the correct depth; if it does not balance, a new trial must be made by assuming a new depth in (2) and repeating (3). (4) Continue these trial and error determinations by reaches until the depths at the given locations have been computed.

Procedure for computing successive distances along the channel to selected depths is: (1) Determine the location of a control section and the depth of flow at that section. (2) Select a depth at the upper or lower end of a reach of length, ℓ, depending on whether the computations are to be carried upstream or downstream. (3) Evaluate s_f and the velocity heads, substitute these values in formula (5.4-36), and compute ℓ. (4) Continue these computations by repeating (1), (2), and (3).

In computing water surface profiles for the design of improved channels, particularly lined channels, by either of these procedures the change in velocity in a reach should be held to a maximum of 15 to 20 percent, that is, neither v_1 nor v_2 should be allowed to vary from the other by more than 15 or 20 percent. This can be done in the trial and error computation of depth by keeping the selected reaches sufficiently short. When distances between depths are being computed by formula (5.4-36) the value of d_1 or d_2, whichever is being selected, can be taken so that neither v_1 nor v_2 is greater or less than the other by more than 15 to 20 percent.

It is recommended that in all cases the computations for surface profiles be carried upstream when the depth of flow is greater than critical and downstream when the depth of flow is less than critical. The first step in the analysis of flow in a channel should be to locate all control sections for the discharges to be investigated. This step sets out the portions of the channel in which depths of flow will be greater or less than critical and spots the stations or sections from which computations should be carried upstream and downstream.

Note that velocity head plus depth, $(v^2 \div 2g) + d$, at sections 1 and 2 is the specific energy at those sections. Inspection of the specific energy diagrams on drawing ES-35 will show that when depth of flow is less than critical, specific energy increases as depth decreases; and when depth of flow is greater than critical, specific energy increases as depth increases. Formula (5.4-36) may be written:

$$\ell = \frac{H_{e2} - H_{e1}}{s_o - s_f}$$

When step computations in a uniform channel of considerable length are to be made, it will often be worthwhile to plot the specific energy diagram for the discharge or discharges to be considered. This diagram may be used as a guide to the selections of depths for successive steps when either formula (5.4-35) or (5.4-36) is being used.

Evaluation of s_f may be made by one of the following formulas:

$$s_f = \frac{n^2\ v_m^{\,2}}{2.2082\ r_m^{4/3}} \tag{5.4-37}$$

$$s_f = \frac{Q^2\ n^2}{2.2082\ a_m^2\ r_m^{4/3}} \tag{5.4-38}$$

$$s_f = \left(\frac{Qn}{Kd_a^{8/3}}\right)^2 \tag{5.4-39}$$

$$s_f = \left(\frac{Qn}{K'\ b^{8/3}}\right)^2 \tag{5.4-40}$$

n = roughness coefficient.

$v_m = \frac{v_1 + v_2}{2}$ = mean velocity in a reach.

$r_m = \frac{r_1 + r_2}{2}$ = mean hydraulic radius in a reach.

$a_m = \frac{a_1 + a_2}{2}$ = mean area of flow in a reach.

$d_a = \frac{d_1 + d_2}{2}$ = average depth in a reach.

b = bottom width of rectangular or trapezoidal channel.

Q = discharge.

K and K' = factors varying with the ratios of certain linear dimensions of cross sections.

In computations for uniform channels, formulas (5.4-39) and (5.4-40) are time savers. This is particularly true of formula (5.4-40) since for a given Q the value of $(Qn \div b^{8/3})^2$ is constant and s_f is obtained by multiplying $(1/K')^2$ for the various values of d_a/b by this constant. King's Handbook contains tables of values of K, K', and $(1/K')^2$ for various types of channels. In nonuniform and natural channels s_f may be computed by formula (5.4-37) or (5.4-38). When a high degree of accuracy is not required, s_f may be obtained from the alignment chart, drawing ES-34, by entering the chart with the appropriate values of v_m, r_m, and n.

A general guide to the analysis of flow conditions in the cases most commonly dealt with in channel design is given by drawing ES-38.

4.7.3 Examples - Uniform Flow. In the majority of cases we will know, or will have selected, the type of cross section and the roughness coefficient and will want to determine either discharge, velocity, channel dimensions, or slope. Methods of solving a number of practical problems are illustrated by examples. The following table summarizes the factors known and to be determined in the various examples; Q and v are discharge and velocity; b, T, d, and z are channel dimensions (see drawing ES-33).

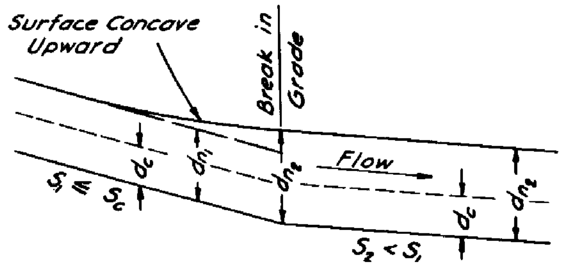
Q	v	b or T	d	z	s	n	Type of Channel	Example No.
0	0	X	X	X	X	X	Trapezoidal	1
0	0		X	X	X	X	Triangular	2
X	0	X	0		X	X	Rectangular	3
X	0	X	0	X	X	X	Trapezoidal	4
X	0		0	X	X	X	Triangular	5
X	0	X	0	X	X	X	Parabolic	6
0	0	X	X	X	X	X	Trapezoidal	7
X	0	0	X	X	X	X	Trapezoidal	8
X	0	0	X		X	X	Parabolic	9
X	X	0	0	X	X	X	Trapezoidal	10
X	X	0	0		X	X	Rectangular	11
X	X		0	0	X	X	Triangular	12
X	X	0	0		X	X	Parabolic	13

X - Known

0 - To be determined

HYDRAULICS: SURFACE PROFILES IN UNIFORM CHANNELS

A

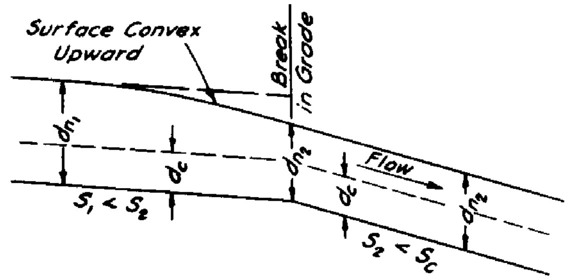


Surface profile is straight, and uniform flow exists downstream from the break in grade. Surface profile immediately upstream from the break in grade is concave upward and is asymptotic to uniform flow surface.

Flow is retarded and sub-critical

Determine the concave upward surface profile by computing upstream from the break in grade starting with the normal depth corresponding to s_2.

B

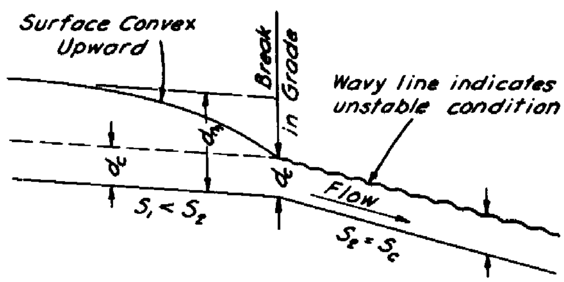


Surface profile is straight, and uniform flow exists downstream from the break in grade. Surface profile immediately upstream from the break in grade is convex upward and is asymptotic to uniform flow surface.

Flow is accelerated and sub-critical

Determine the convex upward surface profile by computing upstream from the break in grade starting with the normal depth corresponding to s_2.

C

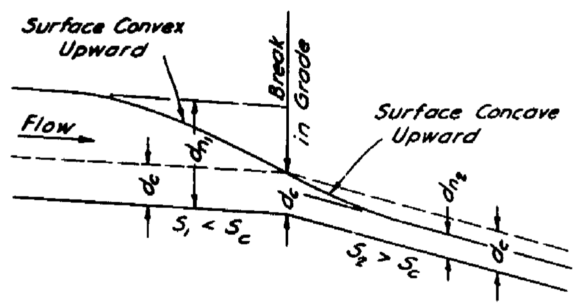


Surface profile is straight, and uniform flow exists downstream from the break in grade. Surface profile immediately upstream from the break in grade is convex upward and is asymptotic to uniform flow surface.

Flow is accelerated and changes from sub-critical to critical flow. This case can occur for only one discharge for a given channel cross-section, slope and roughness coefficient.

Determine the convex upward surface profile by computing upstream from the break in grade starting with the normal depth corresponding to s_2 which is also d_c.

D



Surface profile immediately downstream from the break in grade is concave upward and asymptotic to the uniform flow surface. Surface profile immediately upstream from the break in grade is convex upward and asymptotic to the uniform flow surface.

Flow is accelerated and progresses from sub-critical through critical to super-critical.

Determine the concave upward surface profile by computing downstream from the break in grade, starting with the critical depth. Determine the convex upward surface profile by computing upstream from the break in grade, starting with the critical depth.

NOTE: SURFACE PROFILES ILLUSTRATED ARE BASED ON THE ASSUMPTION THAT THE REACHES OF s AND s_2 ARE SUFFICIENTLY LONG TO PRODUCE UNIFORM FLOW, THUS THE HORIZONTAL SCALE MUST BE VISUALIZED AS BEING GREATLY CONDENSED.

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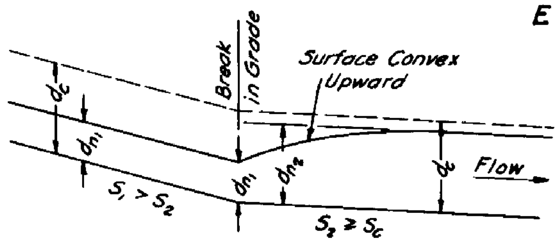
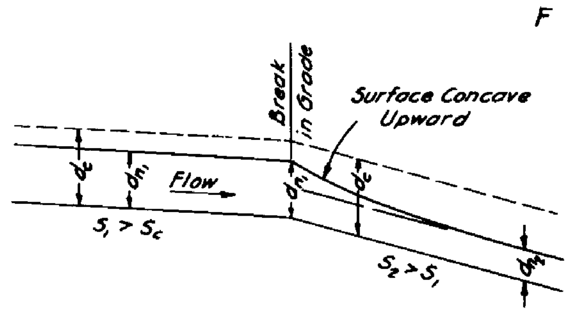
HYDRAULICS: SURFACE PROFILES IN UNIFORM CHANNELS





Surface profile immediately downstream from the break in grade is convex upward and asymptotic to the uniform flow surface. Surface profile is straight, and uniform flow exists throughout the reach upstream from the break in grade.

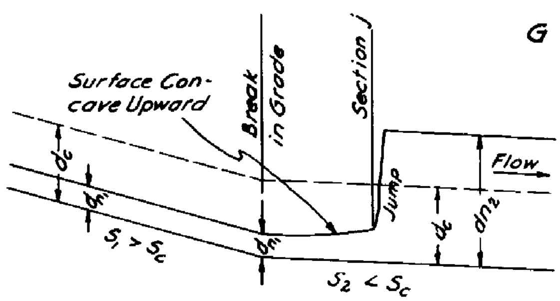
Flow is retarded and super-critical.

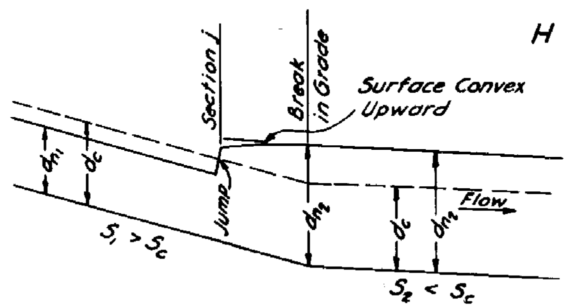
Determine the convex upward surface profile by computing downstream from the break in grade, starting with the normal depth of s_1.

Surface profile immediately downstream from the break in grade is concave upward and asymptotic to the uniform flow surface. Surface profile is straight, and uniform flow exists throughout the reach upstream from the break in grade.

Flow is accelerated and super-critical.

Determine the concave upward surface profile by computing downstream from the break in grade, starting with the normal depth of s_1.





Surface profile is straight, and uniform flow exists downstream from the jump. Surface profile immediately downstream from the break in grade is concave upward. Surface profile is straight, and uniform flow exists upstream from the break in grade.

Flow is retarded and changes abruptly from super-critical to sub-critical.

Surface profile is straight, and uniform flow exists downstream from the break in grade. Surface profile immediately upstream from the break in grade is convex upward. Surface profile is straight, and uniform flow exists immediately upstream from the jump.

Flow is retarded and changes abruptly from super-critical to sub-critical.

The criterion to determine whether the jump occurs downstream or upstream from the break in grade is:

$$a_1 \bar{y}_1 + \frac{Q^2}{ga_1} > a_2 \bar{y}_2 + \frac{Q^2}{ga_2}$$

$$a_1 \bar{y}_1 + \frac{Q^2}{ga_1} < a_2 \bar{y}_2 + \frac{Q^2}{ga_2}$$

See 4.6.1 for nomenclature

Determine the concave upward surface profile by computing downstream from the break in grade, starting with the normal depth corresponding to s_1.

The location of the jump is at the section j within the reach containing the concave upward surface profile satisfying the relation

$$a_j \bar{y}_j + \frac{Q^2}{ga_j} = a_2 \bar{y}_2 + \frac{Q^2}{ga_2}$$

Determine the convex upward surface profile by computing upstream from the break in grade, starting with the normal depth corresponding to s_2.

The location of the jump is at the section j within the reach containing the convex upward surface profile satisfying the relation

$$a_1 \bar{y}_1 + \frac{Q^2}{ga_1} = a_j \bar{y}_j + \frac{Q^2}{ga_j}$$

NOTE: SURFACE PROFILES ILLUSTRATED ARE BASED ON THE ASSUMPTION THAT THE REACHES OF s_1 AND s_2 ARE SUFFICIENTLY LONG TO PRODUCE UNIFORM FLOW, THUS THE HORIZONTAL SCALE MUST BE VISUALIZED AS BEING GREATLY CONDENSED.

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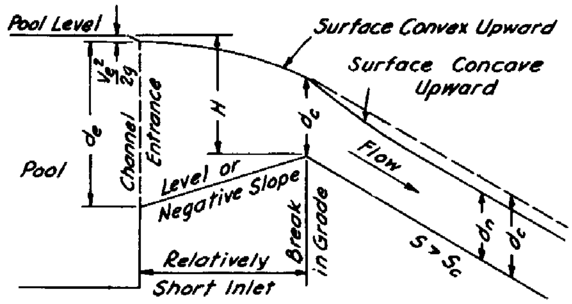
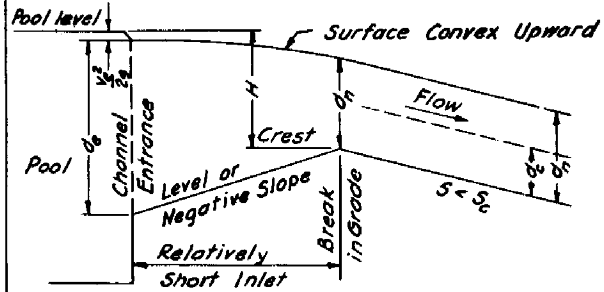
J



Surface profile immediately downstream from the break in grade is concave upward and asymptotic to the uniform flow surface. Surface profile immediately upstream from the break in grade is convex upward.

Flow is accelerated and progresses from sub-critical through critical to super-critical.

K



Surface profile is straight, and uniform flow exists downstream from the break in grade. Surface profile immediately upstream from the break in grade is convex upward.

Flow is accelerated and sub-critical.

General solution for the discharge at a given pool elevation: Each discharge over the crest has its own pool elevation which may be determined. 1st - Determine whether flow conditions are shown by J or K. 2nd - Find two trial discharges; one which has a pool elevation slightly higher and the other slightly lower than the actual pool elevation, H. 3rd - Interpolate for the correct discharge between the two trial discharges.

1. Establish, closely, whether the downstream slope $s <$ or $> s_c$ by:
   - A. Solve for a test discharge Q_t, assuming no loss of head from the channel entrance to the crest when the flow over the crest is critical.
     (Q_t is chosen to balance the equation $d_c + \frac{Q_t^2}{2ga_c^2} = H$; values of d_c and a_c are the critical quantities corresponding to Q_t at the crest.)
   - B. For Q_t, compute s_c. If $s > s_c$, proceed on the left under J; if $s < s_c$, proceed on the right under K.

J

2. A. Take $Q_1 = Q_t$ for first trial discharge.
   - a. Compute, for the discharge Q_1, the convex upward surface profile to the channel entrance starting with the critical depth d_c at the crest; value of d_c is critical corresponding to Q_1.
   - b. Obtain the pool level by adding the velocity head and depth at the channel entrance. The pool elevation, H_1 for the trial discharge, Q_1, will be found to be higher than the actual pool elevation, H.

   B. Select the second trial discharge $Q_2 < Q_1$ such that its pool elevation H_2 is slightly less than H. (Determine H_2 by the same method as stated for H_1 in 2.A.a. and b.)
3. Interpolate for Q, the correct discharge lying between the trial discharges Q_1 and Q_2, given H, H_1, H_2, and observe whether $s > s_c$ for Q; if $s < s_c$, solve by method under K.
4. If $s > s_c$, determine the convex upward surface profile to the channel entrance by computing upstream from the break in grade starting with the critical depth, d_c, corresponding to Q.
5. Determine the concave upward surface profile by computing downstream from the break in grade starting with d_c corresponding to Q.

K

2. A. Solve for the first trial discharge Q_1 by assuming no loss of head from the channel entrance to the crest when the flow over the crest is normal depth for Q_1 downstream from the break in grade. (Q_1 is chosen to balance the equation $d_n + \frac{Q_1^2}{2ga_n^2} = H$; values of d_n and a_n are the normal quantities corresponding to Q_1 on the slope, s.)
   - a. Compute, for the discharge Q_1, the convex upward surface profile to the channel entrance starting with d_n at the crest, d_n being the normal depth corresponding to Q_1 on the slope, s.
   - b. Obtain the pool level by adding the velocity head and depth at the channel entrance. The pool elevation, H_1 for the trial discharge, Q_1, will be found to be higher than the actual pool elevation, H.

   B. Select the second trial discharge $Q_2 < Q_1$ such that its pool elevation H_2 is slightly less than H. (Determine H_2 by the same method as stated for H_1 in 2.A.a. and b.)
3. Interpolate for Q, the correct discharge, lying between the trial discharges Q_1 and Q_2, given H, H_1, H_2, and observe whether $s < s_c$ for Q; if $s > s_c$, solve by method under J.
4. Determine the convex upward surface profile through the inlet by computing upstream from the break in grade, starting with the normal depth, d_n, corresponding to Q in the channel downstream from the break in grade.

NOTE: SURFACE PROFILES ILLUSTRATED ARE BASED ON THE ASSUMPTION THAT THE REACH OF s IS SUFFICIENTLY LONG TO PRODUCE UNIFORM FLOW, THUS THE HORIZONTAL SCALE MUST BE VISUALIZED AS BEING GREATLY CONDENSED. THE INLET MAY BE A NON-UNIFORM CHANNEL.

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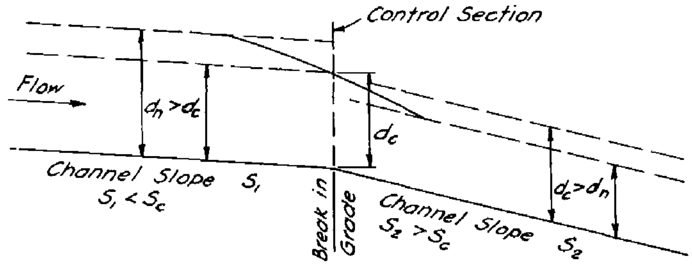
CONTROL SECTIONS

Definition of control section: A flow section at which, for a discharge or a range of discharge, there is a fixed relation between stage or depth of flow and discharge. These conditions are most commonly met at those sections where critical depth occurs. Control section, as used here, does not mean control of discharge. Examples are: (1) a weir; (2) a cross-section of a channel at which the depth of flow is critical, thus establishing that,

$$Q_c = 5.67\ b d_c^{3/2} \quad \text{and} \quad Q_c = d_c^{3/2} \sqrt{\frac{g(b + z d_c)^3}{b + 2 z d_c}}$$

for rectangular and trapezoidal channels respectively. See the subsection 4.5 on Critical Flow.

Control sections are the starting points, both as to station location and elevation of water surface, from which water surface profiles are computed. Consider the following sketch of a break in grade in a uniform channel.



When there is steady flow in a uniform channel with constant roughness coefficient, acceleration or retardation can be caused only by a change in slope. The slopes upstream and downstream from the break in grade determine whether critical depth will occur and, therefore, whether the break in grade is a control section. Refer to paragraph 4.5.4 on Critical Slope and to formula (5.4-24).

$$s_c = \frac{14.56\ n^2\ d_m}{r^{4/3}}$$

s_c = critical slope; d_m = mean depth when flow is critical; r = hydraulic radius corresponding to critical depth; n = roughness coefficient.

Method of Determining Control Sections

Assume a break in grade in a uniform channel as sketched above, to determine whether the break is a control section when the discharge is Q.

1st. Compute d_c corresponding to Q. If the channel is rectangular or trapezoidal, make this computation with the alignment chart, Drawing No. ES-24. When the channel is another form see subsection 4.5 for the formula to be used in computing d_c.

2nd. Compute d_m corresponding to d_c by: d_m = cross-sectional area ÷ width of flow surface. Refer to Drawing No. ES-33, Elements of Channel Sections; also see "King's Handbook", Table 98, "Hydraulic Tables", Tables 4-14 inclusive.

3rd. Compute r corresponding to d_c. "King's Handbook", Tables 97, 101, and 105; "Hydraulic Tables", Tables 4 to 14 incluvive.

4th. Compute the critical slope, s_c,

(a) by formula (5.4-24) given above

(b) or by computing v_c, (Q ÷ area corresponding to d_c, or by appropriate formula in subsection 4.5) entering the alignment chart, Drawing No. ES-34, with v_c, r, and n, and reading s_c.

If $s_1 < s_c < s_2$, d_c occurs at the break in grade and it is a control section. But if $s_1 < s_c > s_2$, or if $s_1 > s_c$, d_c cannot occur and the break in grade is not a control section.

In a given channel s_1 and s_2 are fixed. It is important to remember that d_c and s_c vary with discharge; therefore, it may be found that a break in grade is a control section for some discharges but not all discharges in the operational range.

Revised 4-19-51

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES - 38 SHEET 4 OF 5 DATE 6-18-50

HYDRAULICS: SURFACE PROFILES IN UNIFORM CHANNELS

Example

Given: Concrete lined, rectangular channel, depth 7.5 ft.; width, 20 ft.; $s_1 = 0.0025$; $s_2 = 0.0055$; $n = 0.018$

To determine: If the break in grade between s_1 and s_2 is a control section when the discharge is: 2000, 1000, 500, and 200 c.f.s.

Compute the following tabulated values under instructions given above. In other than rectangular channel, d_m and $d_m/r^{4/3}$ would be required.

Q	Q/b	d_c	r	$r^{4/3}$	$d_c/r^{4/3}$	s_c	Conclusions
2000	100	6.77	4.04	6.43	1.057	0.00498	$s_1 < s_c < s_2$, break is control section.
1000	50	4.27	2.99	4.31	0.993	0.00468	$s_1 < s_c < s_2$, break is control section.
500	25	2.69	2.12	2.72	0.988	0.00467	$s_1 < s_c < s_2$, break is control section.
200	10	1.46	1.27	1.38	1.057	0.00499	$s_1 < s_c < s_2$, break is control section.

$$s_c = 14.56\ n^2 \frac{d_c}{r^{4/3}} = 14.56\ (0.018)^2 \frac{d_c}{r^{4/3}} = 0.00472\ d_c/r^{4/3}$$

STEPS IN ANALYSIS

An analysis of flow in a channel having a number of breaks in grade should be made in the following steps:

1st. Determine the control sections and the depths of flow at those sections for each discharge to be investigated. This sets out the reaches in which the depth of flow will be greater or less than critical and defines the starting points for surface profile computations.

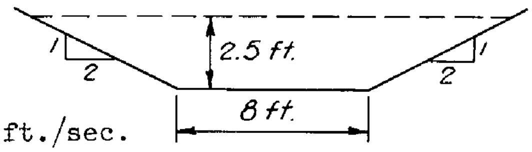
2nd. Compute the surface profiles for each discharge. Carry computations upstream in the reaches where the depth of flow is greater than critical and downstream where the depth of flow is less than critical.

Revised 4-19-51

EXAMPLE 1

Given: Trapezoidal section

$n = 0.02$, $s = 0.006$



To determine: Q in c.f.s. and v in ft./sec.

Solution by formula (5.4-31):

1. $d/b = \frac{2.5}{8} = 0.313$

2. From King's Handbook, table 112, p. 331, K = (by interpolation) 5.96

3. From King's Handbook, table 111, p. 324, $d^{8/3} = 2.5^{8/3} = 11.51$

4. From King's Handbook, table 108, p. 311, or by slide rule, $s^{1/2} = 0.006^{1/2} = 0.0775$

5. $Q = \frac{K}{n} d^{8/3} s^{1/2} = \frac{5.96}{0.02} \times 11.51 \times 0.0775 = 266$ c.f.s.

6. $v = \frac{Q}{a} = 266 \div [(8 \times 2.5)+(2 \times 2.5^2)] = 8.19$ ft./sec.

Solution by formula (5.4-30) without tables or other work aides:

1. $a = (2.5 \times 8)+(2 \times 2.5^2) = 32.5$

2. $p = 8 + 2\sqrt{5^2 + 2.5^2} = 19.18$

3. $r = 32.5 \div 19.18 = 1.695$ $\quad r^{2/3} = 1.422$

4. $s^{1/2} = 0.006^{1/2} = 0.0775$

5. $Q = \frac{1.486}{n} ar^{2/3} s^{1/2} = \frac{1.486}{0.02} \times 32.5 \times 1.422 \times 0.0775 = 266$ c.f.s.

6. $v = \frac{Q}{a} = 266 \div 32.5 = 8.19$ ft./sec.

Solution by alignment chart, drawing ES-34:

1. $r = \frac{a}{p} = 32.5 \div 19.18 = 1.695$

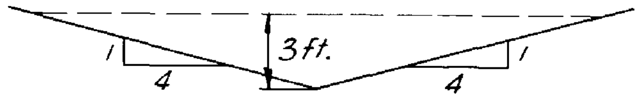
2. Enter chart with $r = 1.695$, $s = 0.006$, $n = 0.02$, and read $v = 8.19$ ft./sec.

3. $Q = av = 32.5 \times 8.19 = 266$ c.f.s.

EXAMPLE 2

Given: Triangular section

$n = 0.025, \quad s = 0.006$



To determine: Q in c.f.s. and v in ft./sec.

Solution by formula (5.4-31) using King's Handbook tables:

1. $d/b = \frac{3.0}{0.0}$ = infinity
2. K = 3.67, table 112, p. 335
3. $d^{8/3} = 3^{8/3} = 18.70$, table 111, p. 324
4. $s^{1/2} = 0.006^{1/2} = 0.0775$, table 108, p. 311
5. $Q = \frac{K}{n} d^{8/3} s^{1/2} = \frac{3.67}{0.025} \times 18.70 \times 0.0775 = 213$ c.f.s.
6. $v = \frac{Q}{a} = 213 \div 36.0 = 5.91$ ft./sec.

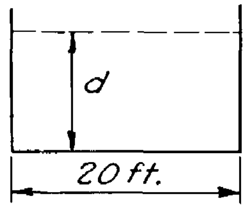
Solution by formula (5.4-1) using King's Handbook tables:

1. d/b = infinity
2. $r = cd = 0.485 \times 3.0 = 1.455$ c, from table 97, p. 296
3. $r^{2/3} = 1.455^{2/3} = 1.284$, from table 109, p. 312
4. $s^{1/2} = 0.006^{1/2} = 0.0775$
5. $v = \frac{1.486}{n} r^{2/3} s^{1/2} = \frac{1.486}{0.025} \times 1.284 \times 0.0775 = 5.91$ ft./sec.
6. $Q = av = 36 \times 5.91 = 213$ c.f.s.

EXAMPLE 3

Given: Rectangular section

$Q = 600$ c.f.s.
$n = 0.025$
$s = 0.0004$



To determine: d in ft. and v in ft./sec.

Solution using formula (5.4-32) in the form $Q = \frac{K'}{n} b^{8/3} s^{1/2}$ with King's Handbook tables:

1. $b^{8/3} = 20^{8/3} = 2950$, table 111, p. 327; $0.0004^{1/2} = 0.02$, table 108, p. 311.

2. $K' = \frac{Qn}{b^{8/3} s^{1/2}} = \frac{600 \times 0.025}{2950 \times 0.02} = 0.254$

3. In table 113, p. 336, column for vertical sides, find $K' = 0.254$, and find, by interpolation, $d/b = 0.448$. Then $d/20 = 0.448$ and $d = 20 \times 0.448 = 8.96$ ft.

4. $v = \frac{Q}{bd} = \frac{600}{8.96 \times 20} = 3.35$ ft./sec.

Solution using formula (5.4-30) and slide rule only:

1. $ar^{2/3} = \frac{Qn}{1.486 s^{1/2}} = \frac{600 \times 0.025}{1.486 \times 0.02} = 505$

2. By assuming values of d, compute $ar^{2/3}$ until one value lower and one higher than 505 is found, as follows:

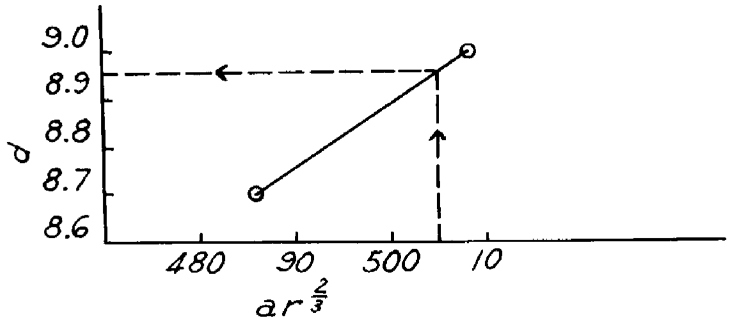
$a = 20d$; $r = \frac{20d}{20 + 2d}$

Trial	d	a	20 + 2d	r	$r^{2/3}$	$ar^{2/3}$
1	8.0	160.0	36.0	4.45	2.71	434
2	8.7	174.0	37.4	4.65	2.79	486
3	9.0	180.0	38.0	4.74	2.82	508

Trials 2 and 3 bracket the value sought.

Example 3 - Continued

3. Plot d versus $ar^{2/3}$ for trials 2 and 3 as follows:



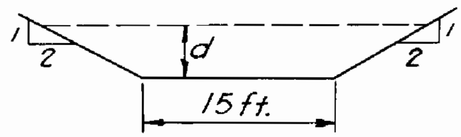
Enter this plot with $ar^{2/3} = 505$ and read d = 8.95 ft.

4. $v = \frac{Q}{bd} = \frac{600}{8.95 \times 20} = 3.35$ ft./sec.

Note: "Hydraulic Tables" makes rapid computations possible in step 2.

EXAMPLE 4

Given: Trapezoidal section



$Q = 300$ c.f.s.
$n = 0.02$
$s = 0.0009$

To determine: d in ft. and v in ft./sec.

Solution by formula (5.4-32) in the form $Q = \frac{K'}{n} b^{8/3} s^{1/2}$ with King's Handbook tables:

1. $b^{8/3} = 15^{8/3} = 1370$, table 111, p. 326
2. $s^{1/2} = 0.0009^{1/2} = 0.03$
3. $K' = \dfrac{Qn}{b^{8/3}\ s^{1/2}} = \dfrac{300 \times 0.02}{1370 \times 0.03} = 0.146$
4. In table 113, p. 336, column for side slopes 2:1, find $K' = 0.150$ for $d/b = 0.23$ and $K' = 0.139$ for $d/b = 0.22$; by interpolation, $d/b = 0.226$ when $K' = 0.146$.
5. $d = 15 \times 0.226 = 3.39$ ft.
6. $v = Q/a = 300 \div [(3.39 \times 15) + (2 \times 3.39^2)] = 4.06$ ft./sec.

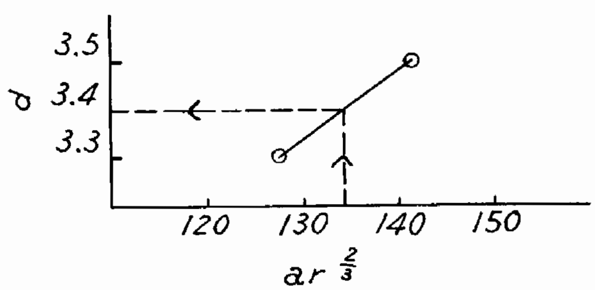
Solution using slide rule and formula (5.4-30):

1. $ar^{2/3} = \dfrac{Qn}{1.486\ s^{1/2}} = \dfrac{300 \times 0.02}{1.486 \times 0.03} = 134.5$
2. By assuming values of d, compute $ar^{2/3}$ for a value higher and lower than 134.5

   $a = 15d + 2d^2$; $p = 15 + 4.47d$; $r = (15d + 2d^2) \div (15 + 4.47d)$

Trial	d	a	15 + 4.47d	r	$r^{2/3}$	$ar^{2/3}$
1	3.0	63.0	28.42	2.21	1.698	107.0
2	3.5	77.0	30.65	2.51	1.847	142.0
3	3.3	71.3	29.76	2.39	1.788	127.5

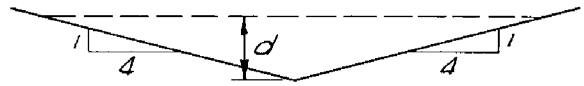
3. Plot d versus $ar^{2/3}$ for trials 2 and 3; where $ar^{2/3} = 134.5$, $d = 3.40$ ft.



4. $v = Q/a = 300 \div [(15 \times 3.4) + (2 \times 3.4^2)] = 300 \div 74.1 = 4.05$ ft./sec.

EXAMPLE 5

Given: Triangular section



$Q = 400$ c.f.s.
$n = 0.025$
$s = 0.005$

To determine: d in ft. and v in ft./sec.

Solution using formula (5.4-31).

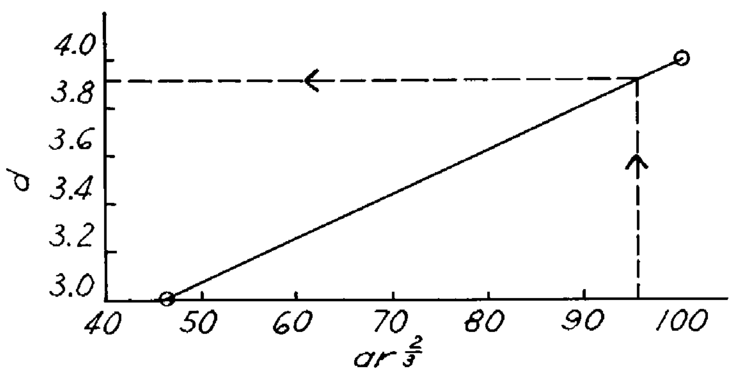
1. d/b = infinity
2. From King's Handbook, table 112, p. 335, col. $z = 4$, $K = 3.67$
3. $d^{8/3} = \dfrac{Qn}{K\ s^{1/2}} = \dfrac{400 \times 0.025}{3.67 \times 0.0707} = 38.55$; $d = 38.55^{3/8} = 3.93$ ft.
4. $v = Q/a = 400 \div (4 \times 3.93^2) = 6.48$ ft./sec.

Solution using formula (5.4-30):

1. $ar^{2/3} = \dfrac{Qn}{1.486\ s^{1/2}} = \dfrac{400 \times 0.025}{1.486 \times 0.0707} = 95.3$
2. Compute the following table assuming d:
   $a = zd^2 = 4d^2$; $p = 2d\sqrt{z^2+1} = 2d\sqrt{17} = 8.24d$; $r = \dfrac{4d^2}{8.24d} = 0.485d$

d	a	r	$r^{2/3}$	$ar^{2/3}$
2	16	0.97	0.98	15.7
3	36	1.455	1.28	46.1
4	64	1.94	1.556	99.6

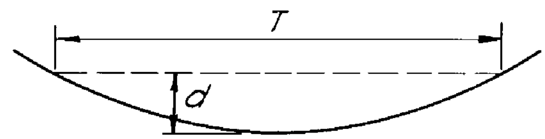
3. Plot $ar^{2/3}$ for $d = 3$ and $d = 4$; enter with $ar^{2/3} = 95.3$ and read $d = 3.92$ ft.



4. $v = Q/a = 400 \div (4 \times 3.92^2) = 6.50$ ft./sec.

EXAMPLE 6

Given: Parabolic section



$Q = 400$ c.f.s.
$n = 0.025$
$s = 0.005$

To determine: d and v when width of water surface T is 10 ft., 20 ft., and 30 ft.

Solution by formula (5.4-32) and King's Handbook tables:

1. $Q = \frac{K'}{n} T^{8/3} s^{1/2}$ or $K' = \frac{Qn}{T^{8/3} s^{1/2}}$

   When T = 10, $K' = \frac{400 \times 0.025}{464 \times 0.0707} = 0.305$

   When T = 20, $K' = \frac{400 \times 0.025}{2950 \times 0.0707} = 0.048$

   When T = 30, $K' = \frac{400 \times 0.025}{8690 \times 0.0707} = 0.0163$

2. From table 118, p. 358, by interpolation

   When K' = 0.305, d/T = 0.746 and d = 10 x 0.746 = 7.46 ft.
   When K' = 0.048, d/T = 0.198 and d = 20 x 0.198 = 3.96 ft.
   When K' = 0.0163, d/T = 0.1002 and d = 30 x 0.1002 = 3.01 ft.

3. a = (2/3)Td v = Q ÷ a

   v = 400 ÷ (2/3 x 10 x 7.46) = 8.03 ft./sec.
   v = 400 ÷ (2/3 x 20 x 3.96) = 7.60 ft./sec.
   v = 400 ÷ (2/3 x 30 x 3.01) = 6.65 ft./sec.

Solution by formula (5.4-30):

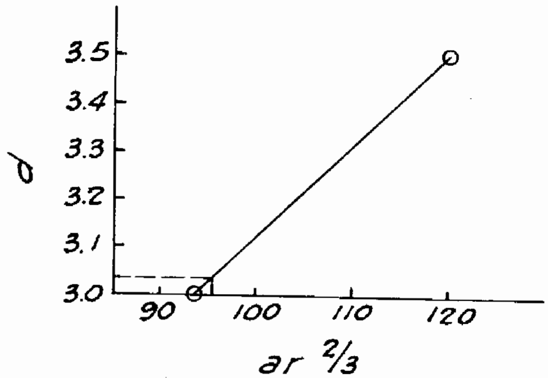
1. $ar^{2/3} = \frac{Qn}{1.486 \times s^{1/2}} = \frac{400 \times 0.025}{1.486 \times 0.0707} = 95.3$

2. Compute the following table for T = 30

   $a = (2/3)Td = 20d$ $r = \frac{T^2\, d}{1.5T^2 + 4d^2} = \frac{900\, d}{1350 + 4d^2}$

d	a	r	$r^{2/3}$	$ar^{2/3}$
2	40	1.318	1.202	48.1
3	60	1.945	1.558	93.5
3.5	70	2.255	1.72	120.4

3. Plot d versus $ar^{2/3}$, enter with $ar^{2/3}$ = 95.3 and read d = 3.02 ft.

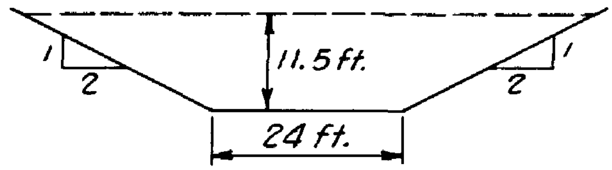


4. $v = \frac{Q}{(2/3)dT}$ = 400 ÷ (20 x 3.02) = 6.63 ft./sec.

EXAMPLE 7

Given: A concrete lined floodway

Trapezoidal section



$n = 0.016$
$s = 0.005$

To determine:

(a) Maximum Q when the freeboard is 1.0 ft., i.e., d = 10.5 ft.

(b) The depth, d, for any Q less than maximum.

Solution by formula (5.4-30):

1. Compute a table as follows:

Columns	1	2	3	4	5	6
	d	r	$r^{2/3}$	a	$ar^{2/3}$	Q
	10.5	6.66	3.54	472.5	1673.	11,000.
	8.5	5.62	3.16	348.5	1101.	7,230.
	6.4	4.48	2.72	235.52	641.	4,210.
	4.4	3.30	2.22	144.32	320.	2,100.
	2.4	1.99	1.58	69.12	109.2	717.
	1.4	1.24	1.15	37.52	43.1	283.
	0	0	0	0	0	0

Col. 1: Assume values of d.

Col. 2: From "Hydraulic Tables", table 10, p. 164.

Col. 3: From "Hydraulic Tables", table 19, p. 294, or "King's Handbook", table 109, p. 312.

Col. 4: From "Hydraulic Tables", table 10, p. 164.

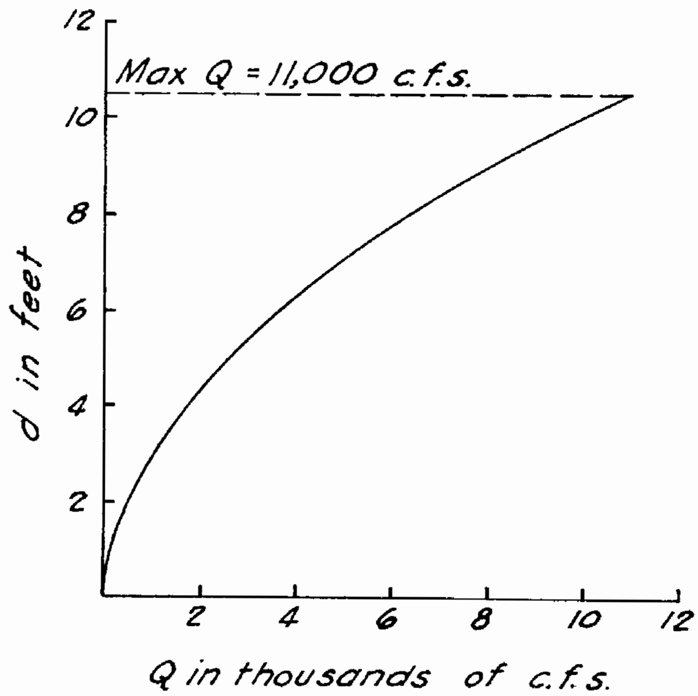
Col. 5: Products of the values in cols. 3 and 4 by slide rule or calculating machine if preferred.

Col. 6: $Q = ar^{2/3} \frac{1.486}{n} s^{1/2}$, $\frac{1.486}{n} s^{1/2} = \frac{1.486}{0.016} \times 0.0707 = 6.57$;

therefore, the values in col. 6 = the values in col. 5 x 6.57.

2. Maximum Q = 11,000 c.f.s.

3. Plot d versus Q, from which d for any given Q may be read.



Solution using formula (5.4-1) and "Hydraulic Tables", taking n as 0.0175 instead of 0.016:

1. Compute a table as follows:

Columns	1	2	3	4	5
	d	r	a	v	Q
	10.5	6.66	472.5	21.26	10,050.
	8.5	5.62	348.5	18.97	6,620.
	6.4	4.48	235.52	16.32	3,840.
	4.4	3.30	144.32	13.31	1,920.
	2.4	1.99	69.12	9.50	656.
	1.4	1.24	37.52	6.96	261.
	0	0	0	0	0

Col. 1: Assume values of d.

Col. 2: Tabulate values of r from table 10, p. 164.

Col. 3: Tabulate values of a from table 10, p. 164.

Col. 4: Tabulate values of v for r, s, and n from table 28, p. 378. Interpolation is required.

Col. 5: Product of the values in cols. 3 and 4 (Q = av).

2. Maximum Q = 10,050 c.f.s.

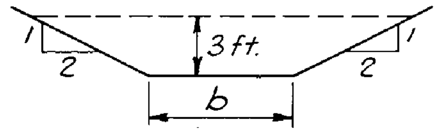
3. From the table developed in step 1, plot a graph of d versus Q, from which d for any given Q may be read.

Note: Cols. 1, 2, and 3 could be tabulated as above, and v in col. 4 computed by the alignment chart, drawing ES-34.

EXAMPLE 8

Given: Trapezoidal section

$Q = 500$ c.f.s.
$s = 0.009$
$n = 0.025$



To determine: b in ft. and v in ft./sec.

Solution using formula (5.4-31) with "King's Handbook" tables:

1. $K = \dfrac{Qn}{d^{8/3}\ s^{1/2}} = \dfrac{500 \times 0.025}{3^{8/3} \times 0.009^{1/2}} = \dfrac{500 \times 0.025}{18.7 \times 0.0949} = 7.04$

2. In table 112, p. 331, column for z = 2-1, find K = 7.08 for d/b = 0.25, and K = 6.87 for d/b = 0.26. Interpolating d/b = 0.252 for K = 7.04

   $b = 3.0 \div 0.252 = 11.90$ ft.

3. $v = \dfrac{Q}{bd + zd^2} = 500 \div [(11.90 \times 3.0)+(2 \times 9)] = 9.31$ ft./sec.

Solution using formula (5.4-30):

1. $ar^{2/3} = \dfrac{Qn}{1.486\ s^{1/2}} = (500 \times 0.025) \div (1.486 \times 0.0949) = 88.8$

2. Compute the following table:

Columns	1	2	3	4	5
	b	a	r	$r^{2/3}$	$ar^{2/3}$
	10	48.00	2.05	1.613	77.4
	12	54.00	2.12	1.650	89.1

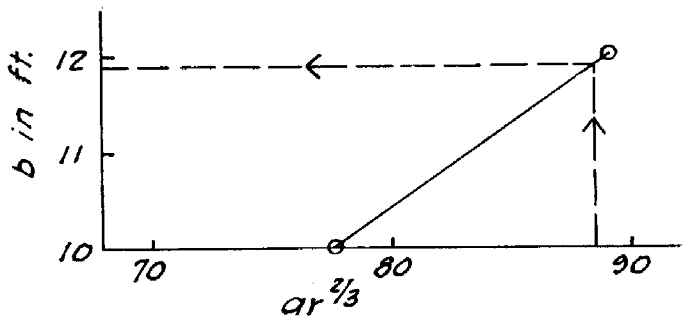
Col. 1: Assume values of b

Cols. 2 and 3: Values of a and r from "Hydraulic Tables", table 10, p. 163. These values may be computed if tables are not available.

Col. 4: From "Hydraulic Tables", table 19, or from "King's Handbook", table 109.

Col. 5: Product of cols. 2 and 4.

3. Plot b versus $ar^{2/3}$, enter with $ar^{2/3} = 88.8$, and read b = 11.90 ft.



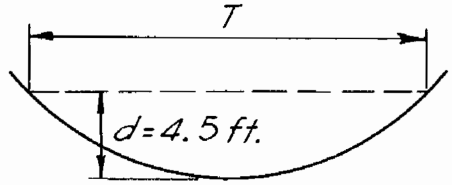
4. $v = \frac{Q}{a} = 500 \div [(11.90 \times 3)+(2 \times 9)] = 9.31$ ft./sec.

Note: The solutions for b and v in rectangular sections are similar to those given above.

EXAMPLE 9

Given: Parabolic section

$Q = 600$ c.f.s.
$n = 0.03$
$s = 0.009$



To determine: T in ft. and v in ft./sec.

Solution by formula (5.4-31) using King's Handbook:

1. $K = \dfrac{Qn}{d^{8/3} \times s^{1/2}} = \dfrac{600 \times 0.03}{55.2 \times 0.09487} = 3.44$

2. In table 117, p. 358, by interpolation, $d/T = 0.2058$

   $T = 4.5 \div 0.2058 = 21.8$ ft.

3. $v = \dfrac{Q}{a} = 600 \div (2/3 \times 21.8 \times 4.5) = 9.2$ ft./sec.

This may also be solved by formula (5.4-30) by:

1. Compute $ar^{2/3} = \dfrac{Qn}{1.486\ s^{1/2}} = 127.7$

2. Holding d constant and assuming T, compute values of $ar^{2/3}$ above and below 127.7.

3. Plotting T versus $ar^{2/3}$ and entering the plot with 127.7 to find T.

4. $v = \dfrac{Q}{2/3\ dT} = \dfrac{600}{3T}$

Cases Where Both d and b or d and T are Required

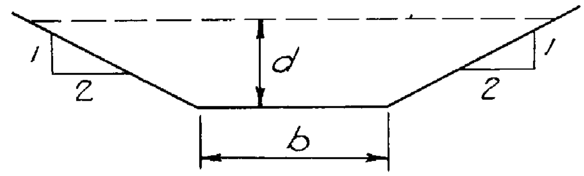
In conservation work, cases are frequently encountered in which Q, n, s, and shape of cross section of a waterway are known but where the allowable velocity is limited by the soils at the waterway site. These cases require the determination of channel dimensions and the solution is tedious if approached on a trial and error basis. The general approach to these problems is given in the following 3 steps:

Step 1. Compute the required r by Manning's formula from the known n, v, and s.

Step 2. Compute the required area by $a = Q/v$.

Step 3. Express a and r in terms of the channel dimensions z, d, b, or T, and solve for the dimensions by these two simultaneous equations. Only two unknowns are involved in any case. In trapezoidal channels z is selected or known, leaving b and d as the required values. In rectangular and triangular channels the two unknowns are b and d, and z and d, respectively. Parabolic channels require that T and d be determined.

EXAMPLE 10



Given: Trapezoidal section

$Q = 500$ c.f.s.
$v = 4.0$ ft./sec.
$n = 0.03$
$s = 0.005$

To determine: b and d in ft.

1. $$r = \left(\frac{vn}{1.486\ s^{1/2}}\right)^{3/2} = \left(\frac{4.0 \times 0.03}{1.486 \times 0.07071}\right)^{3/2} = 1.22$$

2. $a = Q/v = 500 \div 4.0 = 125$ ft.^{2}

3. Completing step 3 (see preceding page) gives a general equation for depth in trapezoidal channels:

$$d = \frac{\frac{x}{r} \pm \sqrt{\left(\frac{x}{r}\right)^2 - 4x}}{2} \qquad (5.4\text{-}41)$$

x varies with z and a and is to be evaluated by:

$$x = \frac{a}{2\sqrt{z^2 + 1} - z} \qquad (5.4\text{-}42)$$

(a) Evaluating x:

$$x = \frac{125}{2\sqrt{5} - 2.0} = \frac{125}{2.47} = 50.61$$

(b) Computing d:

$$d = \frac{\frac{50.61}{1.22} \pm \sqrt{\left(\frac{50.61}{1.22}\right)^2 - (4 \times 50.61)}}{2} = \frac{41.50 \pm 39.00}{2} = \underline{1.25} \text{ or } 40.25$$

(d = 40.25 would obviously not give a practical channel section.)

(c) Compute b:

$$b = \frac{a}{d} - zd = \frac{125}{1.25} - (2 \times 1.25) = 97.50 \text{ ft.}$$

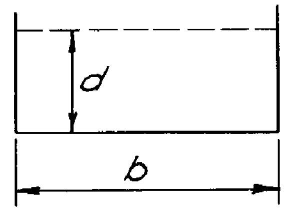
(d) Check:

$$r = \frac{bd + zd^2}{b + 2d\sqrt{z^2 + 1}} = \frac{(97.5 \times 1.25) + 2(1.25)^2}{97.5 + 2.5\sqrt{5}} = \frac{125.13}{103.1} = \underline{1.21} \quad \text{O.K.}$$

EXAMPLE 11

Given: Rectangular section

$Q = 300$ c.f.s.
$v = 5.0$ ft./sec.
$s = 0.005$
$n = 0.03$



To determine: b and d in ft.

1. $r = \left(\frac{vn}{1.486\ s^{1/2}}\right)^{3/2} = \left(\frac{5.0 \times 0.03}{1.486 \times 0.07071}\right)^{3/2} = (1.428)^{3/2} = 1.705$

2. $a = Q/v = 300 \div 5.0 = 60$ ft.^{2}

3. Compute d and b. See step 3, example 10:

$$d = \frac{\frac{x}{r} \pm \sqrt{\left(\frac{x}{r}\right)^2 - 4x}}{2}$$

In rectangular sections $x = \frac{a}{2} = \frac{60}{2} = 30$

$$d = \frac{\frac{30}{1.705} \pm \sqrt{310 - 120}}{2} = \frac{17.6 \pm 13.8}{2} = \underline{1.90} \quad \text{or} \quad 15.70$$

(d = 15.7 is not a practical section.)

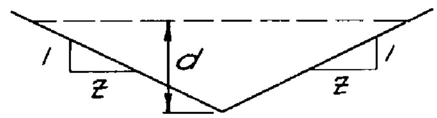
$b = a/d = 60 \div 1.90 = \underline{31.6}$

Check:

$$r = \frac{bd}{b + 2d} = \frac{31.6 \times 1.90}{31.6 + (2 \times 1.90)} = \frac{60}{35.4} = \underline{1.695} \quad \text{O.K.}$$

EXAMPLE 12

Given: Triangular section



$Q = 300$ c.f.s.
$v = 6.0$ ft./sec.
$n = 0.03$
$s = 0.0075$

To determine: d and z

1. $r = \left(\dfrac{vn}{1.486\ s^{1/2}}\right)^{3/2} = \left(\dfrac{6.0 \times 0.03}{1.486 \times 0.0866}\right)^{3/2} = (1.40)^{3/2} = 1.656$

2. $a = Q/v = 300 \div 6.0 = 50$ ft.^{2}

3. This step in the solution for triangular sections is to be accomplished by:

   (a) Entering the graph on drawing ES-39 with the value of a/r^2 and reading z.

   (b) Computing d from $d = \sqrt{\dfrac{a}{z}}$

   Carrying out the solution:

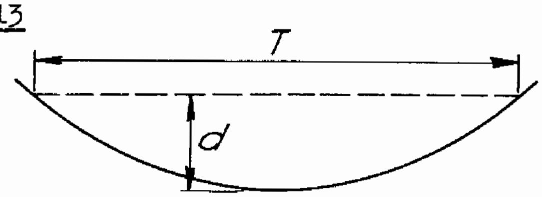
   (a) $a/r^2 = 50 \div (1.656)^2 = 18.25$. From drawing ES-39, $z = \underline{4.30}$

   (b) $d = \sqrt{\dfrac{a}{z}} = \sqrt{\dfrac{50}{4.3}} = \sqrt{11.63} = \underline{3.41}$

   Check:

$$r = \frac{zd}{2\sqrt{z^2 + 1}} = \frac{4.3 \times 3.41}{2 \times 4.41} = \underline{1.66} \quad \text{O.K.}$$

EXAMPLE 13



Given: Parabolic section

$Q = 300$ c.f.s.
$v = 5.0$ ft./sec.
$n = 0.035$
$s = 0.008$

To determine: d and T

1. $$r = \left(\frac{vn}{1.486\ s^{1/2}}\right)^{3/2} = \left(\frac{5.0 \times 0.035}{1.486 \times 0.08944}\right)^{3/2} = (1.317)^{3/2} = 1.51$$

2. $a = Q/v = 300 \div 5.0 = 60$ ft.^{2}

3. This step in the solution of parabolic channels is to be accomplished by:

   (a) Entering the graph on drawing ES-41 with the value of a/r^2 and reading the value of $x = d/T$.

   (b) Computing T from $T = \sqrt{\frac{3a}{2x}}$

   (c) Computing d from $d = xT$

   Carrying out the solution:

   (a) $a/r^2 = 60 \div 1.51^2 = 26.3$. From drawing ES-41, $x = 0.058$.

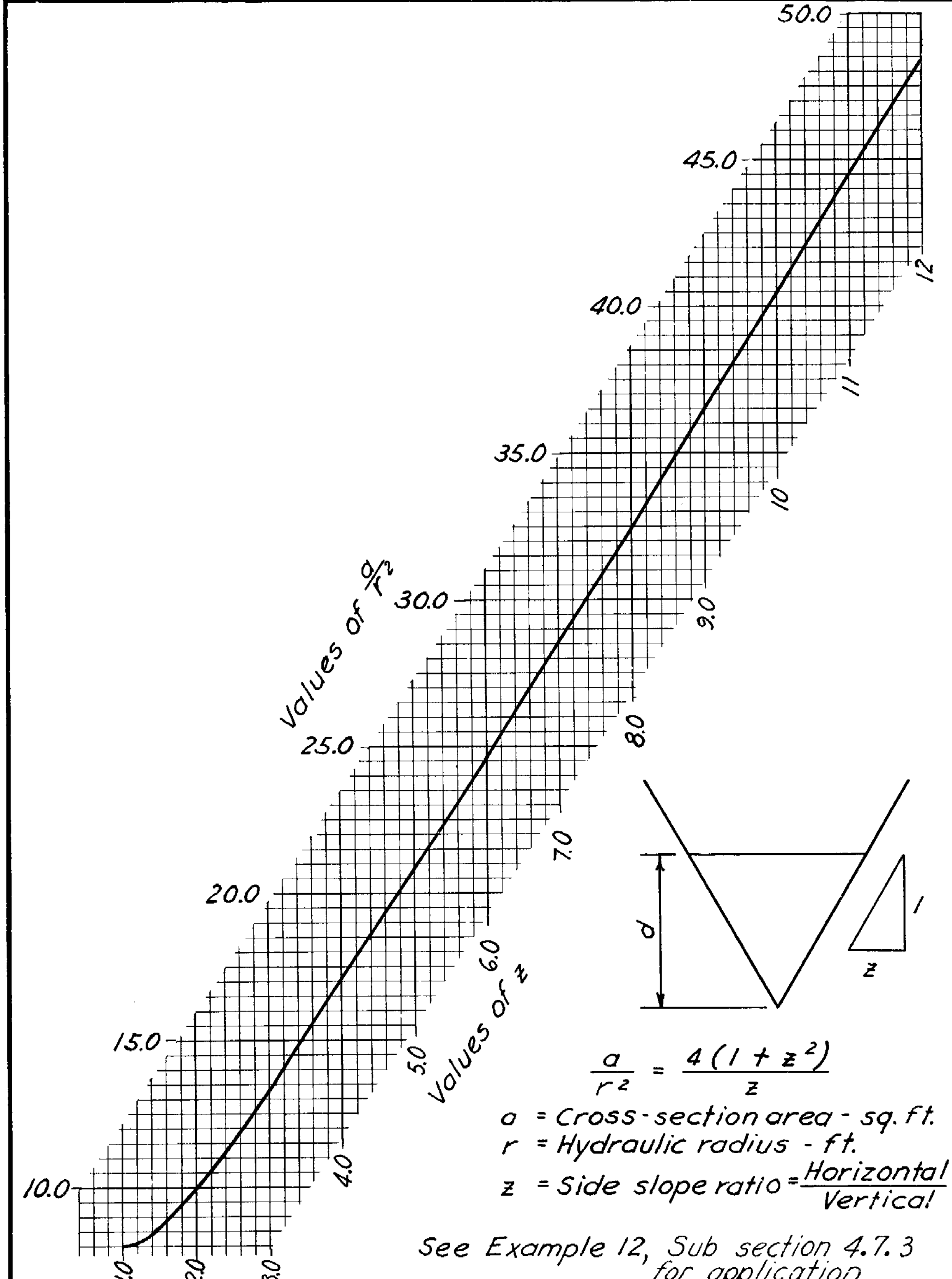
   (b) $T = \sqrt{\frac{3a}{2x}} = \sqrt{\frac{180}{0.116}} = \sqrt{1550} = 39.4$ ft.

   (c) $d = x\ T = 0.058 \times 39.4 = 2.29$ ft.

   Check:

   $$r = \frac{2dT^2}{3T^2 + 8d^2} = \frac{4.58 \times 1550}{(3 \times 1550)+(8 \times 5.24)} = \frac{7110}{4691.9} = \underline{1.51} \quad \text{O.K.}$$

HYDRAULICS: GRAPH FOR DETERMINING SIDE SLOPE z OF A TRIANGULAR CHANNEL WITH Q, v, n, s, GIVEN



$$\frac{a}{r^2} = \frac{4(1+z^2)}{z}$$

a = Cross-section area - sq. ft.

r = Hydraulic radius - ft.

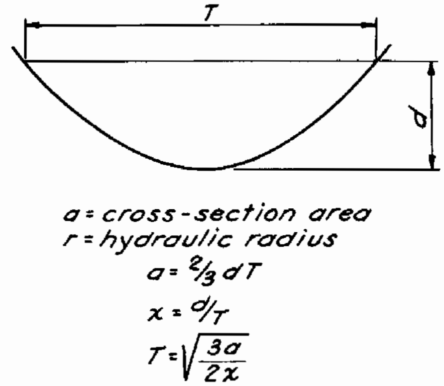
z = Side slope ratio = $\frac{\text{Horizontal}}{\text{Vertical}}$

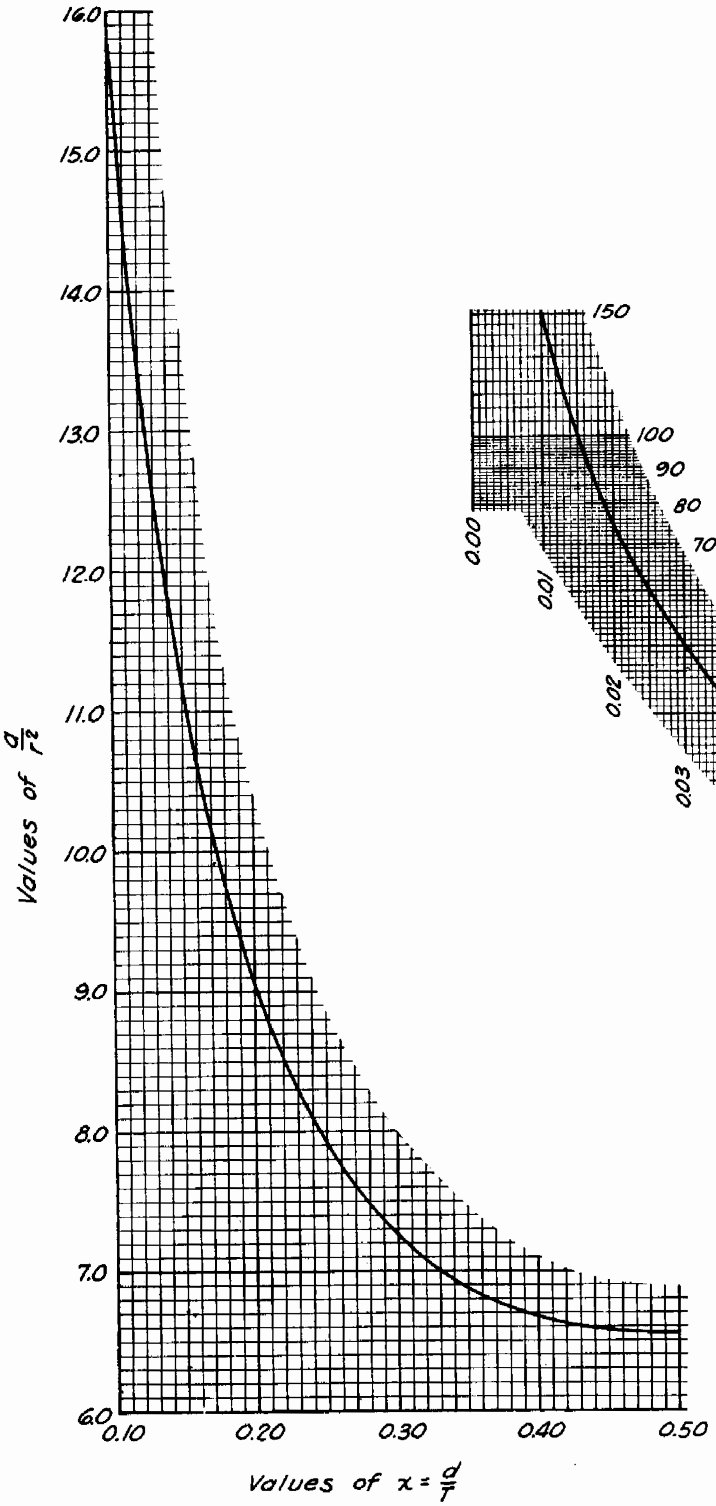
See Example 12, Sub section 4.7.3 for application

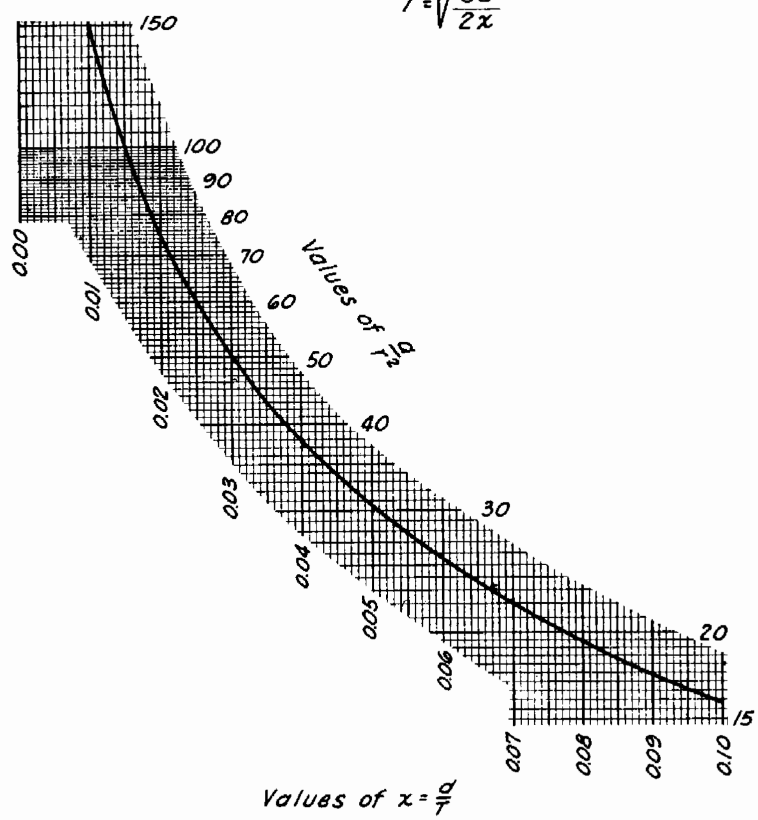
REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES - 39 SHEET 1 OF 1 DATE 7-17-50

HYDRAULICS: GRAPH FOR DETERMINING DIMENSIONS OF A PARABOLIC CHANNEL WITH Q, v, n, s GIVEN

See Example 13, Subsection 4.7.3 for application







REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES-41 SHEET 1 OF 1 DATE 7-21-50

Revised 12-27-50

4.7.4 Examples - Nonuniform Flow in Uniform Channels.

EXAMPLE 1

Given: Concrete, trapezoidal channel, $Q = 1000$ c.f.s., $z = 1.0$, $b = 10$ ft., $n = 0.014$. Station numbers increase in the direction of flow. A break in grade is located at Sta. 33 + 50; the slope upstream from 33 + 50, designated as s_1, is 0.001; the slope downstream from 33 + 50, designated as s_2, is 0.004. A profile of the channel is shown at the end of this example.

To determine: The water surface profile upstream and downstream from the break in grade at Sta. 33 + 50 by computing distance to selected depths using formula (5.4-36).

Solution: (See subsection 4.7.2 and drawing ES-38.)

1. Determine whether Sta. 33 + 50 is a control section.

   (a) Compute critical depth:

   $$Q/b = 1000 \div 10 = 100, \text{ and } z/b = 1 \div 10 = 0.1$$

   From drawing ES-24, $d_c = 5.58$ ft.

   Or using King's Handbook, formula (57), p. 383,

   $$K'_c = \frac{Q}{b^{5/2}} = \frac{1000}{10^{5/2}} = \frac{1000}{316.2} = 3.162$$

   From table 125, p. 436, by interpolation, when $K'_c = 3.162$, $d_c/b = 0.558$ and $d_c = 10 \times 0.558 = 5.58$.

   (b) Compute critical slope: $s_c = 14.56 \dfrac{n^2\, d_m}{r^{4/3}}$

   $$d_m = \frac{a}{T} = \frac{(b + d_c)d_c}{b + 2d_c} = \frac{(10 + 5.58)5.58}{10 + (2 \times 5.58)} = 4.11$$

   $$r = \frac{(b + d_c)d_c}{b + 2d\sqrt{2}} = \frac{(10 + 5.58)5.58}{10 + (2.83 \times 5.58)} = 3.37$$

   $$s_c = \frac{14.56 \times 0.014^2 \times 4.11}{3.37^{4/3}} = \frac{0.0117}{5.05} = 0.00232$$

   $0.001 < 0.00232 < 0.004$; i.e., $s_1 < s_c < s_2$; therefore, Sta. 33 + 50 is a control section.

   Or enter alignment chart, drawing ES-34, v_c, r, n, and read s_c. Use values r and v_c corresponding to d_c.

2. Compute the normal depths (uniform flow) on the slopes upstream and downstream from 33 + 50. This is not necessary, but knowing these depths, facilitates the selections of reaches and trial depths in the computation of the surface profile.

   Use formula (5.4-32) in the form $K' = \dfrac{Qn}{b^{8/3}\ s^{1/2}}$ and King's Handbook, table 113, p. 336.

   (a) For s = 0.001, $K' = \dfrac{1000 \times 0.014}{10^{8/3}\ s^{1/2}} = \dfrac{0.03018}{0.001^{1/2}} = 0.955$

   1:1 side slopes, $d/b = 0.698$, $d = 6.98$

   (b) For s = 0.004, $K' = 0.03018 \div 0.004^{1/2} = 0.477$

   $d/b = 0.482$, $d = 4.82$

   Upstream from 33 + 50 the depth increases from $5.58(d_c)$ to 6.98; downstream the depth decreases to 4.82. See Example 4, subsection 4.7.3 for other methods of determining normal depth.

3. Compute the water surface profile downstream and upstream from Sta. 33 + 50 by computing distances to selected depths using formula (5.4-36).

$$\ell = \frac{\left(\frac{v_2^2}{2g} + d_2\right) - \left(\frac{v_1^2}{2g} + d_1\right)}{s_o - s_f}$$

The table on the following page lists the computations which are carried downstream and upstream because depths are respectively less than and greater than critical.

(a) Cols. 5, 6, and 7 record velocity, velocity head, and specific energy corresponding to the selected depths in col. 4.

(b) The differences in specific energy at successive sections are tabulated in parentheses. This is the numerator of formula (5.4-36).

(c) Computations for the friction slope, s_f, between two successive sections are tabulated in cols. 8, 9, and 10. Friction slope is evaluated by the equation,

$$s_f = \left[(Qn) \div b^{8/3}\right]^2 \quad (1 \div K')^2$$

where $(1/K')^2$ is determined for the average depth (d_a) between successive sections. Tabulated values for $(1/K')^2$ corresponding to d_a/b are taken from "King's Handbook", table 114, pp. 341-356.

(d) Col. 11 is the denominator of formula (5.4-36) and is the difference between friction slope and slope of channel bottom.

(e) Col. 12 lists ℓ, determined by dividing the values in parentheses in col. 7 by the values in col. 11.

Remarks: If reasonably accurate results are to be obtained in computing ℓ by equation (5.4-36), it will frequently be necessary to evaluate $H_e = d + (v^2 \div 2g)$ to the nearest 4th or 5th decimal place. Note that this may be necessary to obtain significant figures in both the numerator and denominator.

COMPUTATIONS FOR WATER SURFACE PROFILE

Example 1 - Subsection 4.7.4

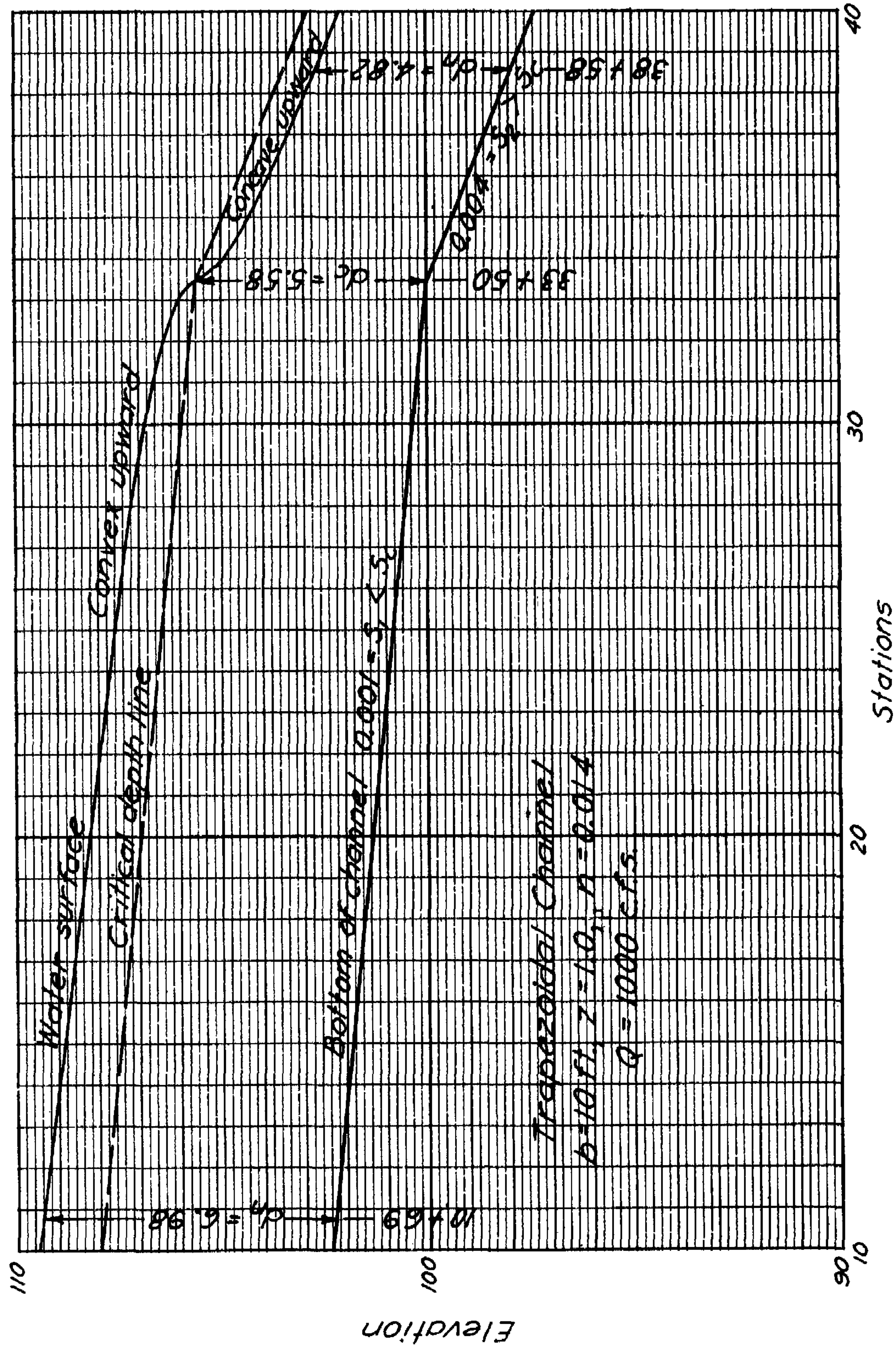
1	2	3	4	5	6	7	8	9	10	11	12
	Elevation										
Sta.	Bottom Channel	Water Surface	d	v	$\frac{v^2}{2g}$	$H_e = d + \frac{v^2}{2g}$	$\frac{d_a}{b}$	$\left(\frac{1}{K'}\right)^2$	s_f	$s_o - s_f$	ℓ
Downstream from Sta. 33 + 50											
33+50	100.00	105.58	5.58	11.503	2.057	7.637					
						(0.011)	0.549	2.71	0.00247	0.00153	7
33+57	99.97	105.37	5.40	12.025	2.248	7.648					
						(0.041)	0.530	3.09	0.00281	0.00119	34
33+91	99.84	105.04	5.20	12.652	2.489	7.689					
						(0.075)	0.510	3.56	0.00324	0.00076	99
34+90	99.44	104.44	5.00	13.333	2.764	7.764					
						(0.103)	0.491	4.09	0.00372	0.00028	368
38+58	97.97	102.79	4.82	13.999	3.047	7.867					
Upstream from Sta. 33 + 50											
33+50	100.00	105.58	5.58	11.503	2.057	7.637					
						−(0.014)	0.569	2.38	0.00217	−0.00117	12
33+38	100.01	105.81	5.80	10.912	1.851	7.651					
						−(0.036)	0.590	2.07	0.00188	−0.00088	41
32+97	100.05	106.05	6.00	10.417	1.687	7.687					
						−(0.0873)	0.615	1.78	0.00162	−0.00062	141
31+56	100.19	106.49	6.30	9.738	1.4743	7.7743					
						−(0.1208)	0.645	1.48	0.00135	−0.00035	345
28+11	100.54	107.14	6.60	9.127	1.2951	7.8951					
						−(0.1916)	0.679	1.22	0.00111	−0.00011	1742
10+69	102.28	109.26	6.98	8.437	1.1067	8.0867					

$Q = 1000$ c.f.s. $\quad z = 1.0$

$b = 10$ ft. $\quad s_1 = 0.001$

$n = 0.014 \quad s_2 = 0.004$

$$\left(\frac{Qn}{b^{8/3}}\right)^2 = 0.00091 \qquad s_f = \left(\frac{Qn}{b^{8/3}}\right)^2 \left(\frac{1}{K'}\right)^2$$



PROFILE

Example 1, Subsection 4.7.4

EXAMPLE 2

Given: A concrete, rectangular channel, Q = 1240 cfs., b = 20 ft., n = 0.014. At sta. 27 + 30 the channel slope changes from 0.018 to 0.0015. The slope of 0.0015 is of sufficient length to assure that uniform flow will occur in the reach downstream from 27 + 30. Bottom of channel elevation at 27 + 30 is 100.00. A profile is shown at the end of this example.

To determine: The location of the hydraulic jump when depth of flow at sta. 27 + 30 is 3.1 ft.

Solution: (See subsection 4.6.2 and drawing ES-38.)

1. Compute the tailwater curve for the flow in the reach downstream from 27 + 30. One of the given conditions is that this reach is long enough to assure the occurrence of uniform flow; therefore, the tailwater curve is a straight line parallel to the channel bottom and at the depth, d_n, for 1240 cfs.

   (a) Compute d_n using formula (5.4-32):

   $$K' = \frac{Qn}{b^{8/3}\ s^{1/2}} = \frac{1240 \times 0.014}{20^{8/3} \times (0.0015)^{1/2}} = \frac{17.36}{114.2} = 0.152$$

   From King's Handbook, table 113, p. 336, by interpolating, find

   $d/b = 0.309$; therefore, $d_n = 20 \times 0.309 = \underline{6.18}$ ft.

2. Compute and plot the water surface profile downstream from 27 + 30 starting with d = 3.10. The jump must occur before the depth has increased to d_c.

   $$d_c = (q^2 \div g)^{1/3} = (Q \div b)^{2/3} \div g^{1/3} = (1240 \div 20)^{2/3} \div 32.2^{1/3} = 4.93$$

   d_c may also be computed from drawing ES-24.

   Use formula (5.4-36):

   $$\ell = \frac{\left(\frac{v_2^2}{2g} + d_2\right) - \left(\frac{v_1^2}{2g} + d_1\right)}{s_o - s_f}$$

   The table on the following page lists the computations:

   Col. 1 starts with sta. 27 + 30 given; other stations are obtained by accumulating the computed distances in col. 12.

   Cols. 2, 3, 5, and 6 are self-explanatory.

COMPUTATION OF WATER SURFACE PROFILE

Example 2 - Subsection 4.7.4

Rectangular Channel:

Q = 1240 c.f.s. n = 0.014
b = 20 ft. s = 0.0015

1	2	3	4	5	6	7	8	9	10	11	12
Sta.	Bottom Channel Elev.	Water Surface Elev.	d	v	$\frac{v^2}{2g}$	$d + \frac{v^2}{2g}$	$\frac{d_a}{b}$	$\left(\frac{1}{K'}\right)^2$	s_f	$s_o - s_f$	ℓ
27+30	100.00	103.10	3.10	20.000	6.219	9.319					
						(0.532)	0.160	295	0.01026	0.00876	61
27+91	99.91	103.21	3.30	18.788	5.487	8.787					
						(0.409)	0.170	246	0.00856	0.00706	58
28+49	99.82	103.32	3.50	17.714	4.878	8.378					
						(0.439)	0.1825	199	0.00694	0.00544	81
29+30	99.70	103.50	3.80	16.316	4.139	7.939					
						(0.351)	0.200	152	0.00529	0.00379	93
30+23	99.55	103.75	4.20	14.762	3.388	7.588					
						(0.194)	0.225	107	0.00372	0.00222	87
31+10	99.43	104.23	4.80	12.917	2.594	7.394					

$$\bar{s}_f = \left(\frac{Qn}{b^{8/3}}\right)^2 \left(\frac{1}{K'}\right)^2 \qquad \left(\frac{Qn}{b^{8/3}}\right)^2 = \left(\frac{1240 \times 0.014}{2950}\right)^2 = 0.0000348$$

Col. 4 starts with d = 3.10 given; other values of d are selected so as to hold the change in velocity, v, about 10 to 15 percent.

s_f is evaluated by formula (5.4-40) which may be written,

$$s_f = \left(\frac{1}{K'}\right)^2 \left(\frac{Qn}{b^{8/3}}\right)^2$$

Col. 7 lists specific energy, with the differences in specific energy at successive sections shown in parentheses.

Col. 8 lists the average of appropriate pairs of depths in col. 4 divided by bottom width.

Col. 9 lists $(1/K')^2$ for the d_a/b values found in King's Handbook, table 114, p. 341.

Col. 10 lists the values in col. 9 multiplied by the constant,

$$\left(\frac{Qn}{b^{8/3}}\right)^2 = 0.0000348$$

Col. 11 is self-explanatory.

Col. 12 is computed by dividing the values in parentheses in col. 7 by the values in col. 11.

3. Compute and plot a curve of sequent depths, i.e., depths after jump corresponding to those shown by the profile computed in step 2. These depths after jump may be computed by formula (5.4-28) or they may be determined by interpolation using "Hydraulic Tables", table 3, p. 16, or "King's Handbook", table 133, p. 444.

Sta.	d_1	v_1	d_2	Bottom Channel Elev.	Sequent Depth Elev.
27 + 30	3.10	20.00	7.37	100.00	107.37
27 + 91	3.30	18.788	7.02	99.91	106.93
28 + 49	3.50	17.714	6.70	99.82	106.52
29 + 30	3.80	16.316	6.26	99.70	105.96
30 + 23	4.20	14.762	5.73	99.55	105.28
31 + 10	4.80	12.917	5.06	99.43	104.49

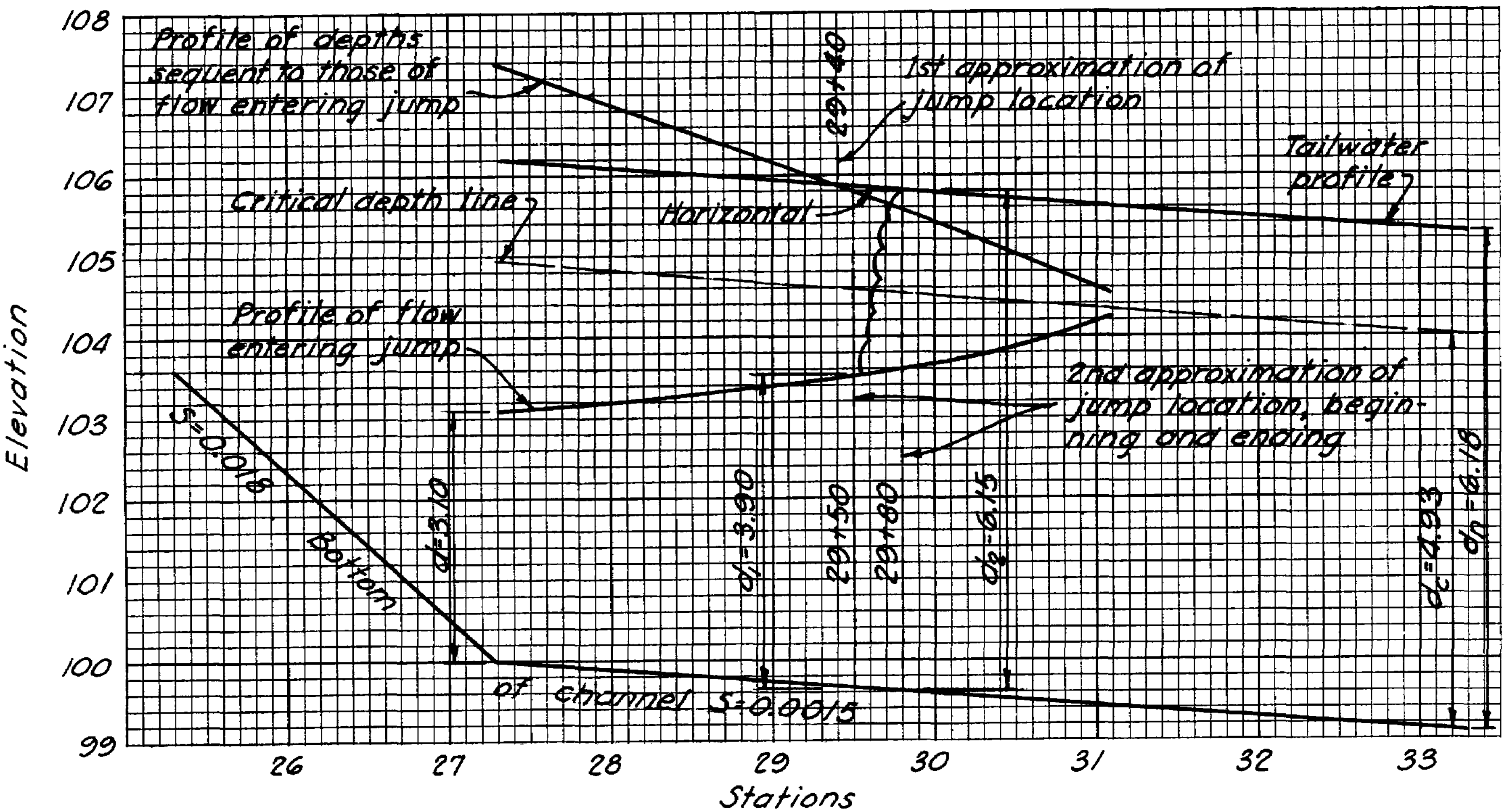
4. The profiles of tailwater, flow entering jump, and sequent depths are plotted at the end of this example. Note that the vertical scale is exaggerated in relation to the horizontal scale.

(a) An approximate location of the jump is sta. 29 + 40 where the profiles of tailwater and sequent depths intersect. On the profile shown on the following page, this is noted as the first approximate jump location.

(b) A second approximation of the jump location, determined by the procedure in step d, subsection 4.6.2, is also shown on the profile. At 29 + 50 the depth of flow entering the jump, d_1, is 3.90; and at 29 + 80 the tailwater depth, d_2, is about 6.15, which is the depth sequent to 3.90, making the length of the jump 30 to 35 ft. or approximately 5 times 6.15.

Note that the accuracy with which water surface profiles may be determined depends on: (1) whether the roughness coefficient, n, accurately represents the condition of the channel; (2) whether the friction losses, s_f, are carefully evaluated. Since the position of the jump depends on the profile of flow entering the jump and the tailwater profile, it is apparent that a minor error in n may result in an appreciable error in the determination of jump location. In short, the estimated position of a jump in a channel not designed to stabilize the jump should be accepted only as an approximation.

Rectangular concrete channel
b = 20 ft.; n = 0.014; Q = 1240 c.f.s.



PROFILE

Example 2, Subsection 4.7.4

4.7.5 Examples - Nonuniform Channels and Natural Channels.

EXAMPLE 1

Given: A rectangular, concrete spillway. Maximum discharge 2000 cfs; roughness coefficient, 0.014. Width of the channel varies uniformly from 30 ft. at entrance to 20 ft. at crest. A profile is shown on the computation sheet.

To determine: The water surface profile from the crest upstream to the spillway entrance and the pool elevation for the discharge 2000 cfs.

Solution: (See subsection 4.7.2) The slope downstream from the crest, sta. 1 + 00, is given as greater than s_c for 2000 cfs; therefore, critical depth occurs at 1 + 00. The water surface profile rises in the upstream direction from the crest through the nonuniform width inlet. The pool elevation for 2000 cfs. is equal to the elevation of the water surface profile at 0 + 00 plus the velocity head at 0 + 00.

1. Compute d_c at the crest, sta. 1 + 00, where the section is 20 ft. wide.

   $$Q/b = 2000 \div 20 = 100 \quad \text{and} \quad z/b = 0$$

   From drawing ES-24, $d_c = 6.77$ ft.

2. Compute the water surface profile from 1 + 00 to 0 + 00 using the energy equation (5.4-35).

   $$\frac{v_1^2}{2g} + d_1 + s_o \ell = \frac{v_2^2}{2g} + d_2 + s_f \ell$$

   (a) Since subscripts 1 and 2 denote upstream and downstream sections respectively, d_2 and v_2 are known for each successive reach as the computations progress upstream.

   (b) The length, ℓ, is selected for each reach.

   (c) In each reach assume d_1 and compute $v_1 = Q/bd_1$. Note that b changes from section to section.

   (d) Evaluate $v_1^2/2g$ and $v_2^2/2g$.

   (e) Evaluate $s_f = \dfrac{v_m^2 \; n^2}{2.2082 \; r_m^{4/3}}$

   $$v_m = \frac{v_1 + v_2}{2} \quad \text{and} \quad r_m = \frac{r_1 + r_2}{2}$$

   (f) Determine whether the assumed d_1 balances equation (5.4-35); if not, take additional trial depths until a balance is obtained.

The table on the following page lists computations made in accordance with the above instructions.

Cols. 1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 18, and 19 are self-explanatory.

Col. 3 lists both d_1 and d_2 for a given reach. For example, in the reach from 1 + 00 to 0 + 90, d_2 = 6.77; and 9.80, 9.66, and 9.67 are trial values of d_1; 9.67 gives a balance for the energy equation, and it becomes d_2 for the reach 0 + 90 to 0 + 80. Values of d_1 giving a balance in the energy equation are marked with subscript 1.

Col. 11 lists values taken from King's Handbook, table 107, p. 309.

Col. 16 lists values of the left-hand member of the energy equation.

Col. 17 lists values of the right-hand member of the energy equation. Note that the values of col. 16 and col. 17 are essentially equal for the trial set of computations for d_1 marked with the subscript 1, thus indicating that the assumed depth is correct.

Note that s_o is negative in this example since the channel slope is adverse to the direction of flow.

COMPUTATIONS FOR WATER SURFACE PROFILE, 1 + 00 to 0 + 00
Example 1 - Subsection 4.7.5

Sta.	b	Depth d	Area a	Velocity $v = \frac{Q}{a}$	$\frac{v^2}{2g}$	$H_e = \frac{v^2}{2g} + d$	$r = \frac{a}{p}$	Elevations: Channel Bottom	Elevations: Surface Profile
1+00	20	6.77	135.40	14.7710	3.3921	10.1621	4.04	1109.20	1115.97
0+90	21	9.80	205.80	9.7182	1.4683	11.2683	5.06		
0+90		9.66	202.86	9.8590	1.5112	11.1712	5.03		
0+90		9.67$_1$	203.07	9.8488	1.5081	11.1781	5.03	1108.20	1117.87
0+80	22	11.20	246.40	8.1169	1.0243	12.2243	5.55		
0+80		11.15$_1$	245.30	8.1533	1.0335	12.1835	5.54	1107.20	1118.35
0+60	24	14.00	336.00	5.9524	0.5509	14.5509			
0+60		13.60	326.40	6.1275	0.5837	14.1837	6.37		
0+60		13.61$_1$	326.64	6.1229	0.5828	14.1928	6.38	1105.20	1118.81
0+00	30	20.00	600.00	3.3333	0.1727	20.1727	8.57		
0+00		20.03$_1$	600.90	3.3283	0.1722	20.2022	8.57	1099.20	1119.23

v_m	r_m	$\frac{1}{2.2082 r_m^{4/3}}$	$(nv_m)^2$	s_f	$s_f \ell$	$s_o \ell$	$\frac{v_2^2}{2g} + d_2 - s_o\ell = H_{e2} - s_o\ell$	$\frac{v_1^2}{2g} + d_1 - s_f\ell = H_{e1} - s_f\ell$
12.24	4.55	0.0601	0.0295	0.00177	0.0177	-1.00	11.1621	11.2506
12.31	4.53	0.0604	0.0296	0.00179	0.0179			11.1533
12.31	4.53	0.0604	0.0297	0.00179	0.0179$_1$			11.1602
8.98	5.29	0.0491	0.0158	0.00078	0.0078	-1.00	12.1781	12.2165
9.00	5.29	0.0491	0.0159	0.00079	0.0079$_1$			12.1756
						-2.00	14.1835	
7.14	5.95	0.0420	0.0100	0.00042	0.0084			14.1753
7.14	5.96	0.0419	0.0100	0.00042	0.0084$_1$			14.1844
4.73	7.47	0.0310	0.00438	0.000135	0.0081	-6.00	20.1928	20.1647
4.73	7.47	0.0310	0.00438	0.000135	0.0081$_1$			20.1941

$\frac{v_e^2}{2g}$ at sta. 0 + 00 = 0.17

Pool elev. = 1119.40

$$\frac{v_2^2}{2g} + d_2 - s_o\ell = \frac{v_1^2}{2g} + d_1 - s_f\ell$$

Q = 2000 cfs
n = 0.014
s_o = -0.10

$$v_m = \frac{v_1 + v_2}{2} \qquad r_m = \frac{r_1 + r_2}{2} \qquad s_f = \frac{(n\ v_m)^2}{2.2082\ r_m^{4/3}}$$

Check: Consider the energy equation between stations 0 + 00 and 1 + 00.

$$\frac{v_1^2}{2g} + d_1 + s_o\ell = \frac{v_2^2}{2g} + d_2 + \Sigma s_f\ell$$

At sta. 0 + 00 (s_o is negative)

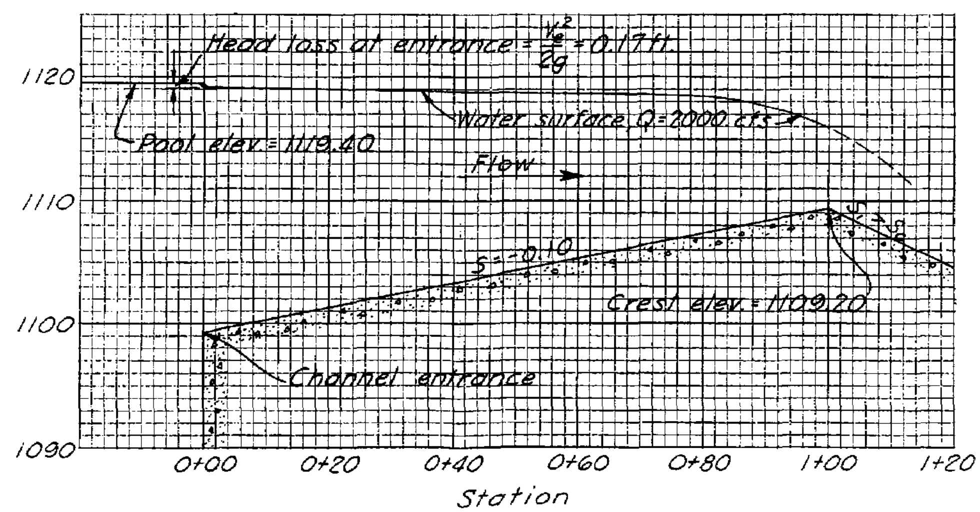
$$\frac{v_1^2}{2g} + d_1 + s_o\ell = 0.1722 + 20.03 - (0.1 \times 100) = 10.2022$$

At sta. 1 + 00

$$\frac{v_2^2}{2g} + d_2 + \Sigma s_f\ell$$

$$= 3.3921 + 6.77 + (0.0179 + 0.0079 + 0.0084 + 0.0081)$$

$$= \underline{10.2024}$$



COMPUTATIONS FOR WATER SURFACE PROFILE, 1 + 00 to 0 + 00

Example 1 - Subsection 4.7.5

Sta.	b	Depth d	Area a	Velocity $v = \frac{Q}{a}$	$\frac{v^2}{2g}$	$H_e = \frac{v^2}{2g} + d$	$r = \frac{a}{p}$	v_m	r_m	$\frac{1}{2.2082 r_m^{4/3}}$	$(nv_m)^2$	s_f	$s_f \ell$	$s_o \ell$	$\frac{v_2^2}{2g} + d_2 - s_o\ell = H_{e2} - s_o\ell$	$\frac{v_1^2}{2g} + d_1 - s_f\ell = H_{e1} - s_f\ell$	Elevations: Channel Bottom	Elevations: Surface Profile
1+00	20	6.77	135.40	14.7710	3.3921	10.1621	4.04										1109.20	1115.97
								12.24	4.55	0.0601	0.0295	0.00177	0.0177	-1.00	11.1621	11.2506		
0+90	21	9.80	205.80	9.7182	1.4683	11.2683	5.06											
								12.31	4.53	0.0604	0.0296	0.00179	0.0179			11.1533		
0+90		9.66	202.86	9.8590	1.5112	11.1712	5.03											
								12.31	4.53	0.0604	0.0297	0.00179	0.0179_1			11.1602		
0+90		9.67_1	203.07	9.8488	1.5081	11.1781	5.03										1108.20	1117.87
								8.98	5.29	0.0491	0.0158	0.00078	0.0078	-1.00	12.1781	12.2165		
0+80	22	11.20	246.40	8.1169	1.0243	12.2243	5.55											
								9.00	5.29	0.0491	0.0159	0.00079	0.0079_1			12.1756		
0+80		11.15_1	245.30	8.1533	1.0335	12.1835	5.54										1107.20	1118.35
														-2.00	14.1835			
0+60	24	14.00	336.00	5.9524	0.5509	14.5509												
								7.14	5.95	0.0420	0.0100	0.00042	0.0084			14.1753		
0+60		13.60	326.40	6.1275	0.5837	14.1837	6.37											
								7.14	5.96	0.0419	0.0100	0.00042	0.0084_1			14.1844		
0+60		13.61_1	326.64	6.1229	0.5828	14.1928	6.38										1105.20	1118.81
								4.73	7.47	0.0310	0.00438	0.000135	0.0081	-6.00	20.1928	20.1647		
0+00	30	20.00	600.00	3.3333	0.1727	20.1727	8.57											
								4.73	7.47	0.0310	0.00438	0.000135	0.0081_1			20.1941		
0+00		20.03_1	600.90	3.3283	0.1722	20.2022	8.57										1099.20	1119.23

Q = 2000 cfs
n = 0.014
s_o = -0.10

$$v_m = \frac{v_1 + v_2}{2} \qquad r_m = \frac{r_1 + r_2}{2} \qquad s_f = \frac{(n\,v_m)^2}{2.2082\, r_m^{4/3}}$$

$$\frac{v_2^2}{2g} + d_2 - s_o\ell = \frac{v_1^2}{2g} + d_1 - s_f\ell$$

$\frac{v_e^2}{2g}$ at sta. 0 + 00 = 0.17

Pool elev. = 1119.40

Check: Consider the energy equation between stations 0 + 00 and 1 + 00.

$$\frac{v_1^2}{2g} + d_1 + s_o\ell = \frac{v_2^2}{2g} + d_2 + \Sigma\, s_f\ell$$

At sta. 0 + 00 (s_o is negative)

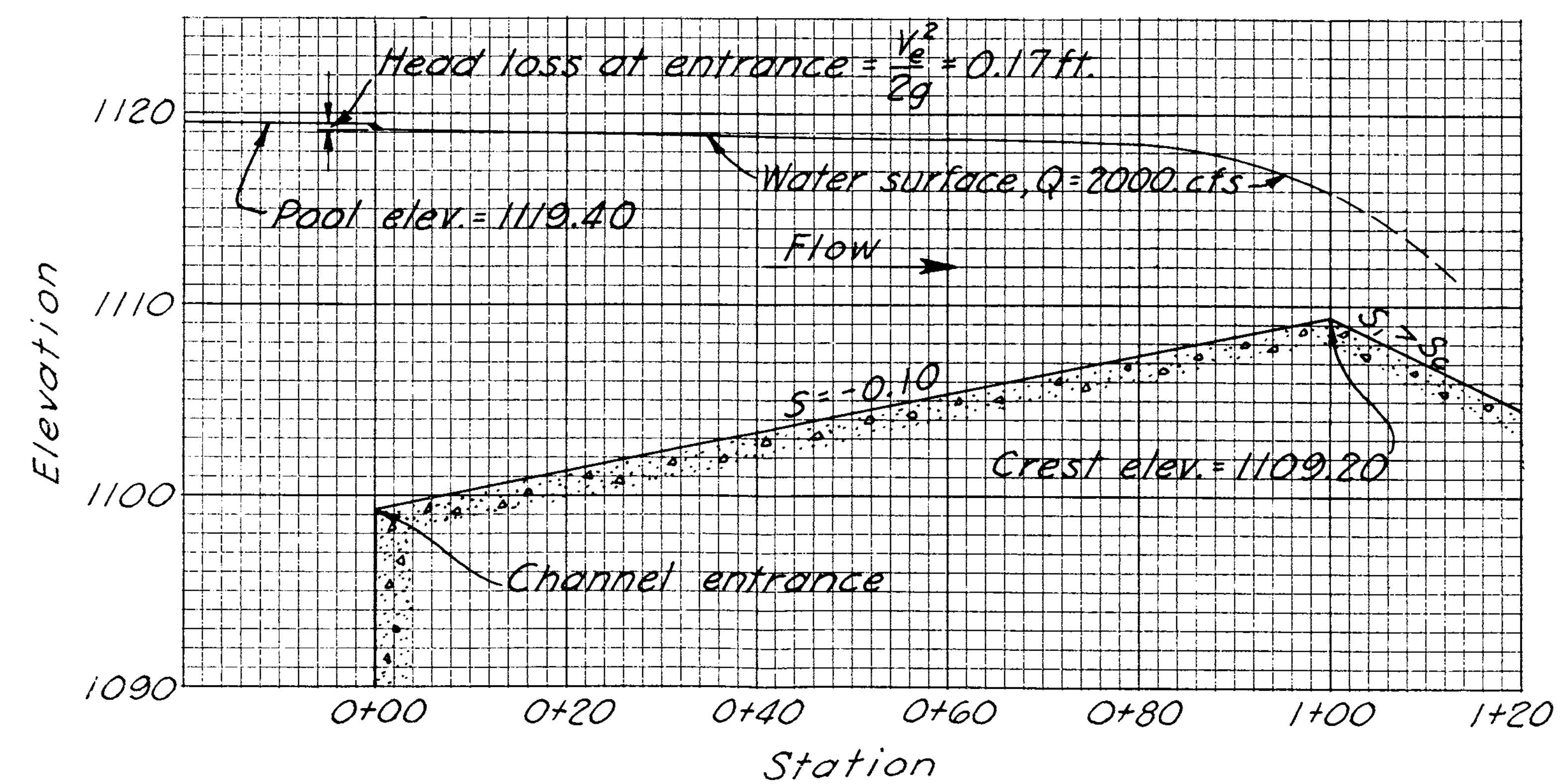
$$\frac{v_1^2}{2g} + d_1 + s_o\ell = 0.1722 + 20.03 - (0.1 \times 100) = 10.2022$$

At sta. 1 + 00

$$\frac{v_2^2}{2g} + d_2 + \Sigma . s_f\ell$$

$$= 3.3921 + 6.77 + (0.0179 + 0.0079 + 0.0084 + 0.0081)$$

$$= \underline{10.2024}$$



EXAMPLE 2

In dealing with natural channels, it is common practice to use an average slope and an average cross section for a reach in the open channel formulas. A further simplification is normally made by the assumption that the variations in velocity head may be neglected. The decision as to whether this latter assumption may be applied in a given case should be made by the engineer from a consideration of the conditions involved. If the changes in velocity head cannot be neglected, the water surface profile may be computed by the method illustrated in Example 1 of this subsection.

When velocity head may be neglected, the method illustrated by the following example results in a direct solution. This method requires that stream profile, cross sections, and roughness coefficients be obtained by field surveys and that water surface elevations at the lower end of the stretch of stream for given discharges be known or determinable. The method is taken from: "Graphical Calculation of Backwater Eliminates Solution by Trial" by Francis F. Escoffier, Engineering News Record, June 27, 1946.

Given: A portion of a natural stream is shown in plan and profile on following pages. Cross sections are available at stations 23 + 00, 26 + 00, 29 + 00, 33 + 50, 38 + 00, 42 + 00, and 46 + 00. The cross section at station 38 + 00, which is typical, is shown on a following page.

To determine: The water surface profile for Branches E and E-2 for the three sets of conditions stated in the following table:

Sets of Conditions	Discharges - cfs			Elev. Water Surface 46 + 00
	Branch E-1	Branch E-2	Branch E	
A	1800	600	2400	1181.50
B	1200	400	1600	1180.75
C	600	200	800	1180.20

Theory:

The energy equation (5.4-35) between sections at the upper and lower ends of a reach is:

$$\frac{v_1^2}{2g} + d_1 + s_o\ell = \frac{v_2^2}{2g} + d_2 + s_f\ell$$

Assuming the change in velocity heads to be negligible results in:

$$\frac{v_1^2}{2g} = \frac{v_2^2}{2g} \; ; \quad \text{then} \quad d_1 - d_2 + s_o\ell = s_f\ell$$

From Manning's formula:

$$Q = \frac{1.486}{n} ar^{2/3} s_f^{1/2}, \text{ and since } s_f = \frac{h_f}{\ell}$$

$$\frac{h_f}{\ell} = Q^2 \left(\frac{n}{1.486\ ar^{2/3}}\right)^2$$

Placing $\left(\frac{n}{1.486\ ar^{2/3}}\right)^2 = P$ results in $h_f = Q^2 P \ell$

$$\therefore\ d_1 - d_2 + s_o \ell = h_f = Q^2 P \ell$$

h_f = loss of head due to friction.

Since the value of P varies with n and water surface elevation at each cross section, assume an average for a reach as valid.

$P = 1/2\ (P_2 + P_1)$, in which

P_2 = the value of P at the lower end of a reach.

P_1 = the value of P at the upper end of a reach.

$$\therefore\ h_f = 1/2\ (P_2 + P_1) Q^2 \ell \text{ or } \frac{h_f}{P_2 + P_1} = \frac{Q^2 \ell}{2}$$

Values of P_2 and P_1 versus water surface elevation are plotted as in fig. 5.4-3.

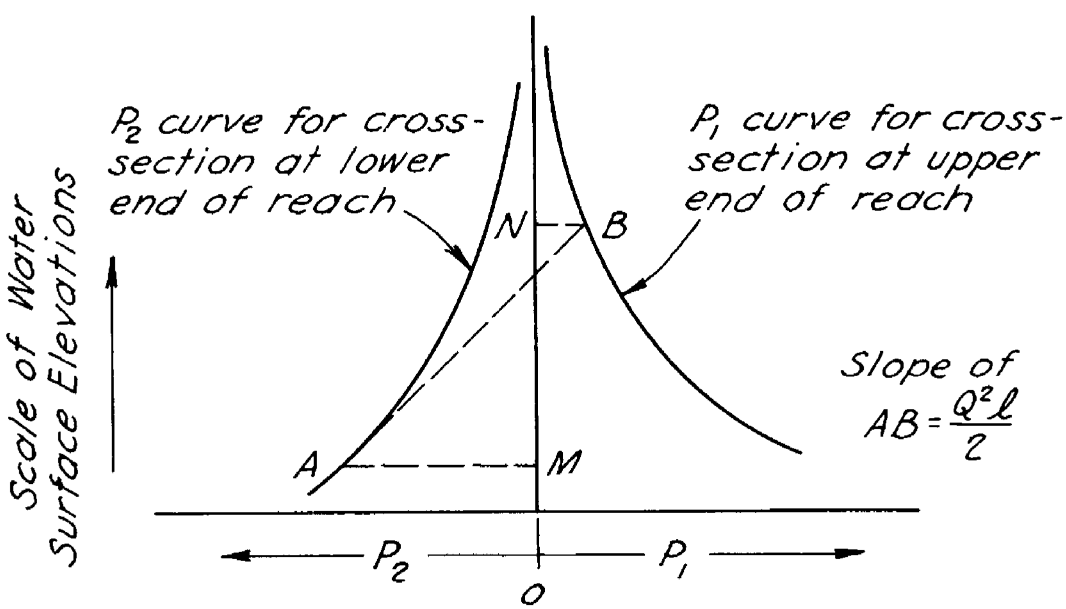


FIG. 5.4-3

If the known elevation of the water surface for a discharge, Q, at the lower end of a reach, point M, is projected to A, the elevation of the water surface at the upper end of the reach, point N, may be determined by constructing AB on the correct slope. By inspection,

$$MA = P_2 \ , \ NB = P_1 \ , \text{ and the slope of AB is } \frac{MN}{MA + NB} = \frac{h_f}{P_2 + P_1}$$

Since $h_f \div (P_2 + P_1) = (Q^2 \ell) \div 2$, the slope at which AB is to be drawn is $(Q^2 \ell) \div 2$.

Solution:

1. Compute the P values for each cross section. Table I lists the computations for P at one foot intervals of elevation in the cross section at 38 + 00; other cross sections would be treated in a similar manner. The cross section is given on a following page. Column headings in the table are self-explanatory.

   (a) Values of $(1.486/n)ar^{2/3}$ are first computed in each part of the cross section and totaled at each elevation for the entire cross section.

   In a cross section not subdivided,

   $$P = \left(\frac{n}{1.486 \ ar^{2/3}}\right)^2 \quad \text{and} \quad \frac{1}{\sqrt{P}} = \frac{1.486}{n} ar^{2/3}$$

   In a subdivided cross section,

   $$P_T = \frac{1}{\left(\frac{1}{\sqrt{P_a}} + \frac{1}{\sqrt{P_b}} + \frac{1}{\sqrt{P_c}} + \text{---}\right)^2} = \frac{1}{\left(\sum_{1}^{n} \frac{1}{\sqrt{P_i}}\right)^2}$$

   P_T = the P value for an entire cross section at any elevation.

   P_a, P_b, P_c, etc. = the P values for parts of the cross section at any elevation.

   $n \geq 2$, the total number of parts of a cross section at any elevation.

   (b) This method makes it possible to apply different n values in selected subdivisions of a channel cross section.

   (c) In a subdivided part of a channel only the perimeter of the channel in contact with water should be used in computing r. That portion of the perimeter where water is in contact with water should be excluded in computing r.

2. Plot the values of P for each cross section as P_2 and P_1 as shown on the work sheet. P values for the cross section at 38 + 00 are plotted twice as P_2 values for reach 3 on the left side of the sheet, and as P_1 values for reach 2 on the right. P values at 42 + 00, 33 + 50, 29 + 00, and 26 + 00 are also plotted as both P_2 and P_1 values for appropriate reaches. The P values for the cross sections at 46 + 00 and 23 + 00 are plotted only once; those at 46 + 00 as P_2 values for reach 1, and those at 23 + 00 as P_1 values for reach 6.

3. Compute values of $Q^2\ell \div 2$ for the reaches and sets of discharge conditions:

Reach				A Condition		B Condition		C Condition	
No.	From	To	ℓ	Q	$\frac{Q^2\ell}{2} \times 10^{-8}$	Q	$\frac{Q^2\ell}{2} \times 10^{-8}$	Q	$\frac{Q^2\ell}{2} \times 10^{-8}$
1	46+00	42+00	400	2400	11.52	1600	5.12	800	1.28
2	42+00	38+00	400	2400	11.52	1600	5.12	800	1.28
3	38+00	33+50	450	2400	12.96	1600	5.76	800	1.44
4	33+50	29+00	450	2400	12.96	1600	5.76	800	1.44
5	29+00	26+00	300	600	0.54	400	0.24	200	0.06
6	26+00	23+00	300	600	0.54	400	0.24	200	0.06

4. Determine the three water surface profiles using: (a) the given water surface elevations at 46 + 00; (b) the $Q^2\ell \div 2$ values tabulated above; (c) the work sheet on which the P curves are plotted.

Examine the set-up of the work sheet. The scales of P_2, P_1, and water surface elevation and the plotting of the P curves will be readily understood. Note the slope scale, i.e., the scale of $Q^2\ell \div 2$ values, on the right and the reference point near the bottom center. The value of the slope scale unit in relation to the reference point is determined as follows:

$$\text{slope} = \frac{\text{vertical distance}}{\text{horizontal distance}} = \frac{h_f}{P_2 + P_1} = \frac{Q^2\ell}{2}$$

The vertical distance = 1 scale unit = 1 foot.

Each horizontal unit has the value 1×10^{-9}. The horizontal distance from the reference point to the slope scale = 10 scale units = $10 \times 1 \times 10^{-9}$ = 10^{-8}; therefore, the value of one unit on the slope scale is:

$$\text{slope} = \frac{1}{10^{-8}} = 1 \times 10^{8}$$

To solve the water surface profile for the "C" condition proceed as indicated below.

On the P_2 curve for reach 1, station 46 + 00, locate the point for water surface elevation 1180.20, Q = 800 cfs. From the table of $Q^2\ell \div 2$ values, take 1.28×10^8 for "C" condition, reach 1, and find this value on the slope scale, thus defining a slope line. Draw a line parallel to this slope line from the point of elevation 1180.20 on the P_2 curve for station 46 + 00 to intersect the P_1 curve for reach 1, station 42 + 00. This intersection establishes the water surface elevation, 1181.23, at the upper end of reach 1 and the lower end of reach 2. Project this water surface elevation to the left to the P_2 curve, reach 2, station 42 + 00, and repeat the above procedure for reaches 2 through 6 to complete the water surface profile for "C" condition. Note that the discharge changes to 200 cfs at reach 5, and that this fact has been recognized in the computation of $Q^2\ell \div 2$ values.

Water surface profiles for "A" and "B" conditions are determined in the same manner and the three profiles are plotted on the last page of this example.

Remarks:

This and other methods for determining backwater curves require that the water surface elevation at the lower end of a stretch of stream be known or that it be determined through some type of stage-discharge relationship. In some cases a gaging station or control point makes it possible to meet these requirements, but in most cases they cannot be met directly by determinations at a single section. In such cases a series of several contiguous reaches should be established downstream from the stretch of stream under consideration, and several profiles for any required value of Q based on assumed water surface elevations at the downstream end of the lowest reach should then be computed upstream. These profiles will converge in the upstream direction making it possible to determine or closely approximate the water surface elevations for given discharges at the lower end of the stretch.

An important advantage of this method is that when the P values have been computed and the work sheet constructed, only a short time is required to make the graphical solution for the water surface profile of any discharge.

TABLE I - COMPUTATIONS OF "P" VALUES FOR SECTION AT STA. 38 + 00
Example 2 - Subsection 4.7.5

Elev.	Part a (n = 0.075)					Part b (n = 0.030)					Part c (n = 0.050)				
	a ft^2	p ft	r ft	$r^{2/3}$	$\frac{1.486}{n} ar^{2/3} = \frac{1}{\sqrt{P_a}}$	a ft^2	p ft	r ft	$r^{2/3}$	$\frac{1.486}{n} ar^{2/3} = \frac{1}{\sqrt{P_b}}$	a ft^2	p ft	r ft	$r^{2/3}$	$\frac{1.486}{n} ar^{2/3} = \frac{1}{\sqrt{P_c}}$
1178	0					1.8	13.1	.137	0.266	23.7	0				
1179	0					29.0	33.2	.874	0.914	1310.	0				
1180	0					59.0	35.4	1.665	1.405	4100.	0				
1181	0					93.4	37.6	2.48	1.83	8450.	0				
1182	0					130.2	39.8	3.27	2.20	14200.	0				
1183	3.4	8.6	0.396	0.540	36.4	168.	40.	4.20	2.60	21600.	29.6	58	0.511	0.64	560.
1184	49.4	55.6	0.888	0.924	905.	206.	40.	5.15	2.98	30400.	90.8	65	1.40	1.25	3370.
1185	105.8	60.0	1.762	1.46	3060.	244.	40.	6.10	3.34	40300.	159.2	72	2.21	1.70	8050.
1186	165.4	62.6	2.64	1.91	6250.	282.	40.	7.05	3.68	51400.	232.8	78	2.99	2.07	14300.

Elev.	$\frac{1}{\sqrt{P_T}}$	$\sqrt{P_T} \times 10^6$	$P_T \times 10^{12}$
1178	23.7	42300.	179×10^7
1179	1310.	764.	584,000.
1180	4100.	244.	59,500.
1181	8450.	118.2	14,000.
1182	14200.	70.5	4,970.
1183	22196.	45.15	2,040.
1184	34675.	28.85	832.
1185	51410.	19.48	379.
1186	71950.	13.9	193.

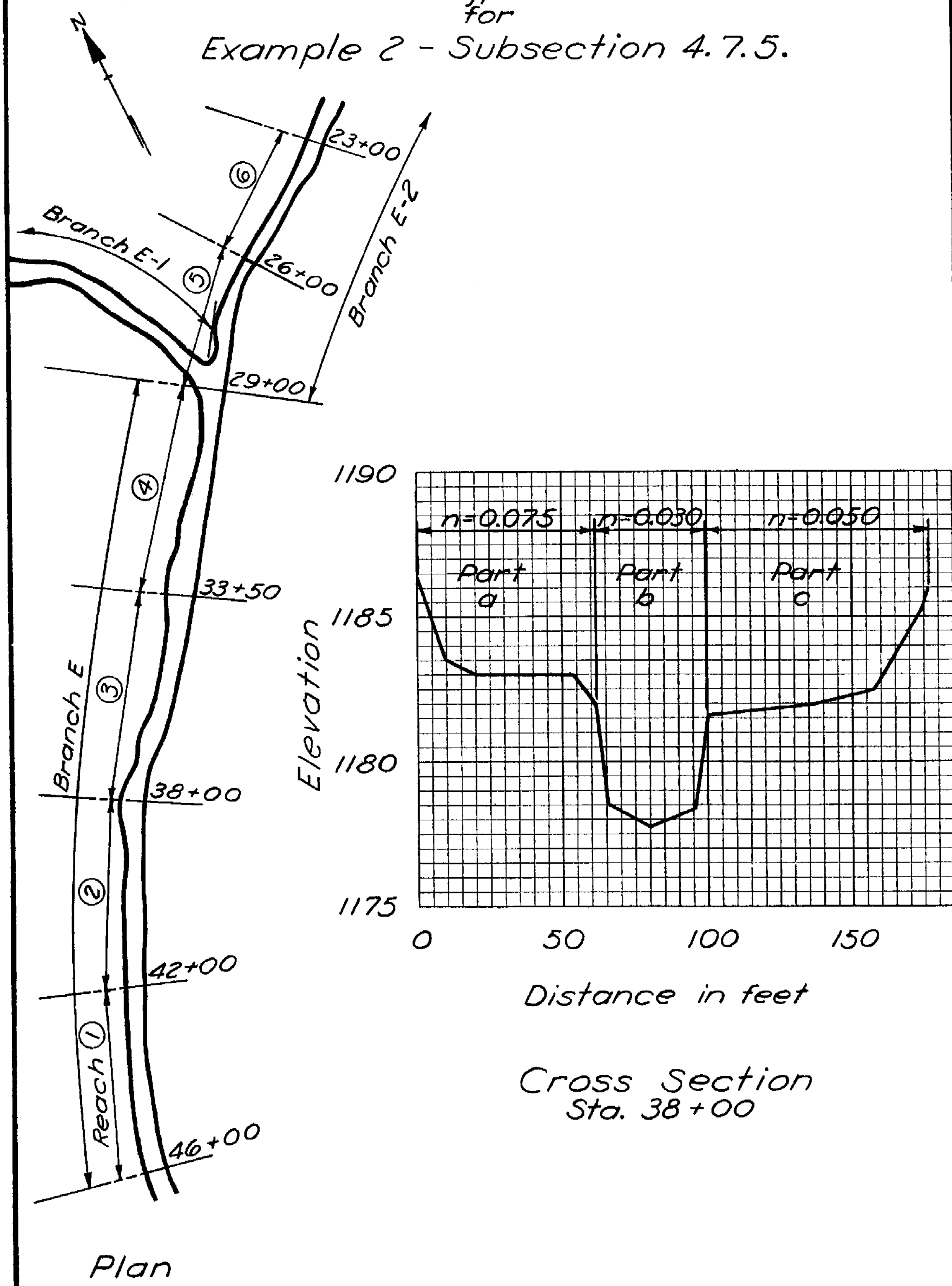
a = area of stream

p = wetted perimeter

r = hydraulic radius

$$\frac{1}{\sqrt{P_T}} = \sum \frac{1.486}{n} ar^{2/3} \text{ at each elevation}$$

Sketch Plan and Typical Cross Section for Example 2 - Subsection 4.7.5.

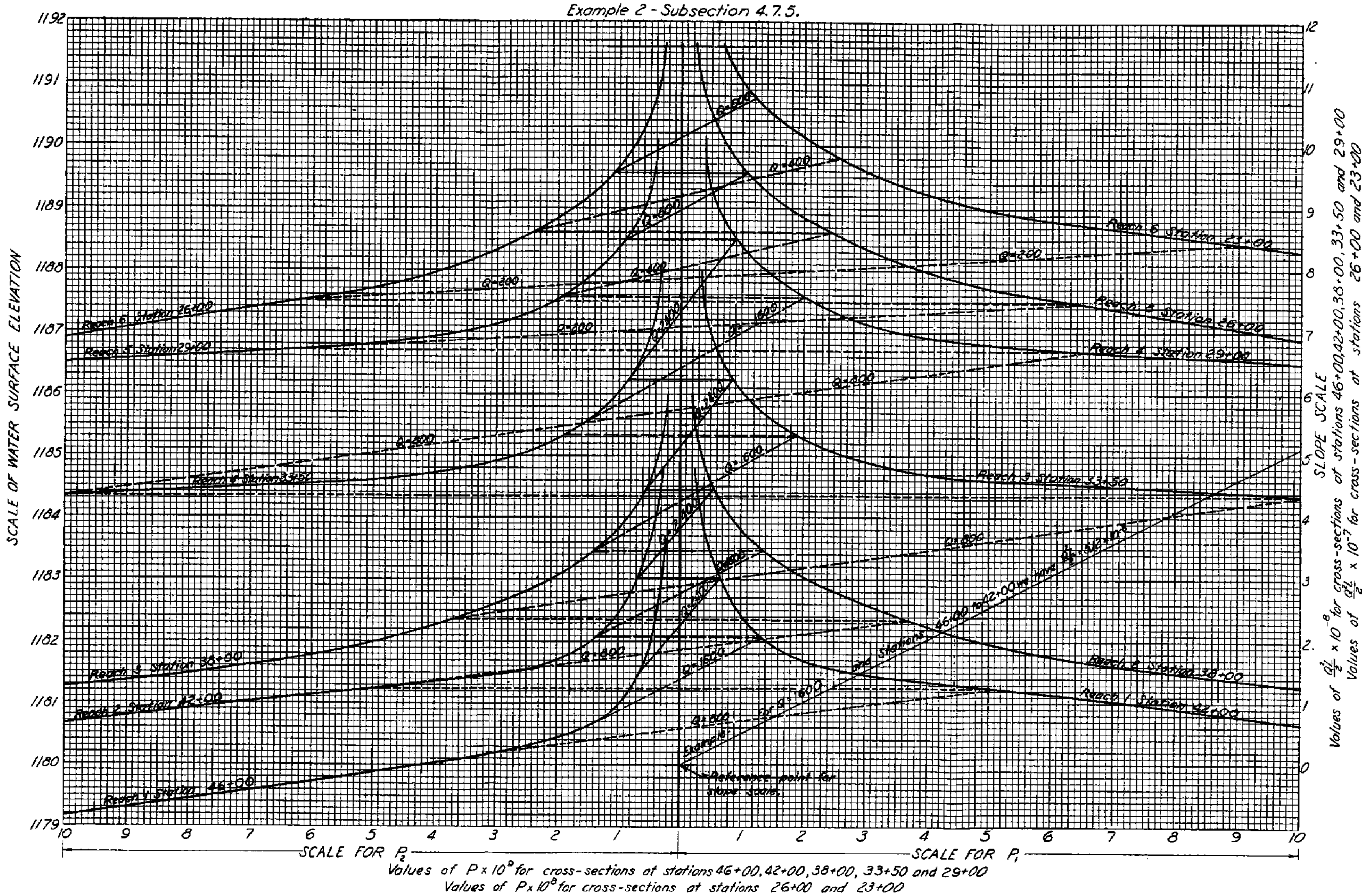


Cross Section
Sta. 38+00

5-4-59

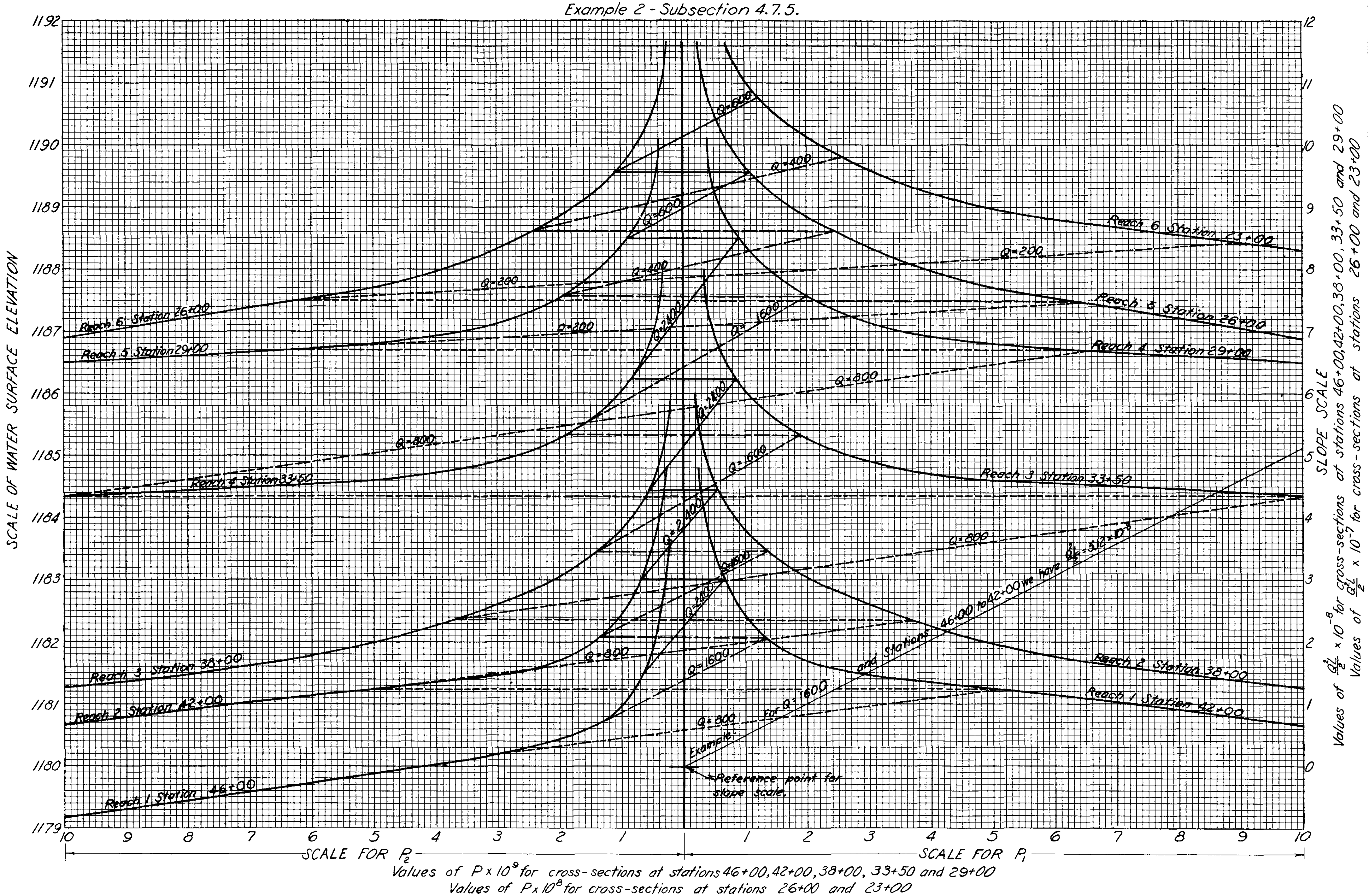
WORK SHEET

Example 2 - Subsection 4.7.5.



WORK SHEET

Example 2 - Subsection 4.7.5.



Water Surface Profiles for Example 2 - Subsection 4.7.5.